Chapter 5
Methods and Methodology

5.1 Introduction

5.1.1 Method

According to Cohen, Manion and Morrison (2000) **method** means "the range of approaches used in educational research to gather data which are to be used as a basis for inference and interpretation, for explanation and prediction" (p. 44). They also say that "the aim of methodology is to help us to understand, in the broadest possible terms, not the products of scientific inquiry but the process itself" (p. 45).

This inquiry started with the teachers' discussion on problems students were encountering while dealing with vectors in Pure Mathematics, mathematical and physical Mechanics. The area of Mechanics has been chosen and exploratory research was conducted. The results of this research helped in developing the preliminary research. This in turn helped in development of the pilot study and then the main study. Cohen, Manion and Morrison (2000) quote Merton and Kendall who, as long ago as 1946, argue that one should try to find a balance between the quantitative and qualitative data and concern oneself "with the combination of both which makes use of the most valuable features of each" (p. 45).

In order to strike the balance this research method draws upon qualitative and quantitative data. The quantitative data, which includes: pre-test, post-test and delayed post test gives an indication to problems which were then scrutinized more thoroughly by collecting the qualitative data gathered during lesson observation and interviews.

The focus of the tests (further described in chapter 6.5.1) has been on finding students' ability:

- of using vectors to solve problems in different mathematical (graphical, coordinates) and physical contexts (displacement, forces);

- to describe their understanding of vectors and vector quantities;
- to think flexibly (using different mode to add vectors: numerical, graphical);
- to think logically instead of depending on their instincts, in particular, solving singular cases;
- to understand that different procedures can produce equivalent outcomes (the same effect).

5.1.2 Methodology

The methodology is partly influenced by the Piagetian view that a consistently made error to a given problem reflects the child's cognitive structure. The errors therefore, recorded from the tests, have been used to give the direction for the interview questioning. The analysis is based upon the students' perspective and their way of functioning with respect to the task rather than upon the logic of the task, which should provide insight into the student's cognition.

Piaget developed the clinical interviewing procedure which later has been developed and used to achieve the analysis by Ginsburg 1981, Swanson et al. 1981. However, this research uses a semi-clinical interviewing technique in which every interview had a common starting point (their responses to the tests). Thereafter the questions followed what they said or what needed to be clarified. The questions have been based only on the topic of vectors as this is the topic through which the research has been done.

The research has been done with two groups (control and experimental), consisting of a total of 34 students (17 in each group). The research was conducted during year 12. All of the students, from the same school, were studying Pure Mathematics with Mechanics. Both groups were using the same text books but had different teachers. The experimental approach has been introduced to the group we will call group A, who were taught by the researcher. The experimental approach was introduced after the pre-test for a period of three lessons. Its aim has been to:

- o help students in reappraisal of the roles of physical action and symbolic manipulation which relates actions having the same effect to mathematically equivalent concepts;
- o encourage students to share ideas and to help them reassess and refine their knowledge;
- o relate different aspects of mathematical theory to different aspects of mathematical applications in Physics.

The experimental approach has been conducted to test researcher's hypothesis that introducing students to the specific embodied experience followed by purposeful plenaries will help these students:

- o to build a better conceptual base to the topic of vectors;
- o sustain the knowledge for a longer period of time;

Graham (the author of the Mechanics books) and Berry, theorized that:

> "Rectification can then take place by providing alternative explanations that the students can see overcome the weaknesses of their original ideas, explaining satisfactorily the situations used to challenge the students' intuitive ideas." (1997, p. 847)

I have adapted this way of challenging students' ideas during plenary discussions and on an individual basis.

As in the Kerslake (1977) experiment, I have based the experimental teaching phase on good teaching practice and 'cognitive instruction' defined by Belmont and Butterfield (1977). "In this model the child's thought processes and the use made of the instruction are monitored as the treatment progresses. The experimenter must observe as directly as possible how the child is thinking while performing a criterion task, having identified the nature of successful reasoning on that task. The important feature of this model is that the researcher's task is to help the child to build up a particular cognitive framework," (Kerslake, 1977, p. 6).

The experimental lessons have been video-recorded in order to observe as much as possible students' reasoning while performing the tasks and to establish if the experimental procedures and plenaries with the whole class help students to establish a particular cognitive framework.

The ethics of research arose from the fact that the students were under 18 years of age and the informed consent has been obtained from the parents of the students who were to be interviewed. As the teacher was conducting the research it was considered important to study Mason's (1994) considerations at the ways of validating such semi-action research.

Mason (1994) distinguishes between three research perspectives: *intraspection* (observing oneself), "*extraspection* (looking from outside), and *interspection* (sharing and negotiating observations with others)" (p. 13). He argues that: "research from the inside can be every bit as systematic and disciplined as traditional (extraspective) research; researching from the inside provides a much-needed balance to traditional research," (p. 2). He also emphasises that methodology of the research done from inside has to be "supported by a consistent epistemology, and that norms for justification and validation of conjectures and assertions are maintained and developed," (p.11) and suggests that "inner research depends on constantly re-validating distinctions and frameworks with colleagues," (p. 11). He writes that: "Distinctions and frameworks which arise from inner research have a domain of validity consisting of the perceiver-researcher, and the situations in which the distinction comes to mind and is found to be informative," (p. 12). According to him, "Inner research recognises that classifications are evidence of sensitivities which are in flux and may alter and develop over time," (p. 14). From my observation some things do change but others such as students' difficulties stay the same. Students who come to study Pure Mathematics with Mechanics come already with their 'conceptual baggage' gained in the previous years. The inner researcher by doing research during more than one year can seek classifications that are robust and transferable from year to year. Inner researchers can also validate their investigations by studying work done by others and look for confirmation of classifications which are being used.

As it has been impossible to run an extraspective research in the full meaning of that word, the research had to be semi-extraspective. During the four years of research it was inevitable that the researcher and other teachers at the school would be affected by the findings at each stage. This was even more probable as the first stage of findings were published during the course of the research (Watson, 2002). The research has therefore become *semi-action research*.

The intraspection perspective plays a very important role when validating data or looking at preliminary studies in comparison with the main study. Mason writes "the educator has noticed, thereby bringing awareness into consciousness and enabling it to inform future practice," (p. 14) which means that conducting research is bound to have an influence on the researcher and therefore on the educator if it is the same person (*intraspection*). However, the researcher/educator has to discuss the findings with his/her colleagues to be able to conduct the research and therefore they are also influenced by the initial assumptions. The tests conducted in the control groups also had some influence on the possible importance of specific ideas on interchanged with other teachers (*interspection*). So to some extent the balance was sustained between the teaching approaches of the experimental group and the control group.

According to Mason, "Outer research seeks classifications that are robust and transferable from researcher to researcher. Taking into consideration Mason's advice on methodology, frequent interviews with colleagues were conducted throughout the research. They served to understand the way students were introduced to vectors in Physics and Mathematics, problems they encountered during studying the topic and teachers' predictions on the outcomes of the tests. The tests were moderated due to comments made by other teachers in order to rationalise the language used, and to establish the questions which would provide valuable data. Talking to the teachers showed the need for asking questions in many different ways and in different contexts. This has given them a better opportunity to show their understanding of a vector and vector addition. The literature involved in the research into vectors and fractions which show many similarities in problems they cause to learners has been studied. The results of this have been presented in chapters 2 and 3.

The influence of teaching through visual methods and a reflective style, specifically concentrated on the equivalence relationships, has been examined to identify possible changes in students' conceptual development.

5.2 Research Design

5.2.1 Sample

The quantitative study involved 17 students in the experimental group A and 17 students in the control group B both from the same school and the same year but taught by different teachers. The group of students who were investigated can be called a *non-probability sample* as it "derives from the researchers targeting a particular group, in the full knowledge" that it might not "represent the wider population," (Cohen, Manion and Morrison, 2000, p. 102).

Purposive sampling has been conducted to choose the students for the interviews. "In purposive sampling, researchers handpick the cases to be included in the sample on the basis of their judgement of their typicality. In this way, they build up a sample that is satisfactory to their specific needs," (Cohen, Manion and Morrison, 2000, p. 103). In this research the sample for the interviews has been chosen on the basis of the error that has been recurring and needed to be investigated further or the students who have answered in a specific way needed to be questioned about their knowledge of different methods.

5.2.2 Triangulation

According to Cohen, Manion and Morrison, "triangular techniques in the social sciences attempt to map out, or explain more fully, the richness and complexity of human behaviour by studying it from more than one standpoint and, in so doing, by making use of both quantitative and qualitative data," (2000, p. 112). This implies a *multi-method approach* to a problem, which attempts to ensure the validation of the research. Denzin (1970) talks also about other types of 'methodological triangulation': *time triangulation, space triangulation, combined levels of triangulation, investigator triangulation and methodological triangulation.*

Time triangulation has been achieved by longitudinal studies which were based on collecting data from the same group of students at different points in the time sequence. The same test three times was given before the course, straight after and the last one, half a year after the course. The time spans throughout the whole year to make sure students do not remember the test too clearly from one time to another. From the experience it seemed that the time span in students' busy life has been enough to make them view the test as new each time.

Theoretical triangulation has been achieved by drawing on alternative theories. The data was analysed and interpreted within the theoretical framework described in the previous chapters. Results of the pre- and post-course questionnaires, together with school-set tests were used to select students for the follow-up interviews.

The students' responses were triangulated with the intentions of the authors of the school text books and the interviews with the teachers on their preferences.

The methodological triangulation was attempted by collecting data through research questionnaires, school tests, exam results, interviews. To support the data triangulation, the results were collected from students of all the abilities and taught by different teachers. There were also efforts made to integrate the quantitative and qualitative techniques to validate the results.

5.2.3 Variables

The term variable is going to be used in an educational context and it will describe the aspects which might vary. For example variables can include methods of answering questions, concept images or levels of performance.

Inner research as Mason (1994) suggests "depends on constantly re-validating distinctions and frameworks with colleagues" (p. 11). Therefore one should seek to obtain results “which are invariant over specific domain of potential variation, and to develop means for validating that invariance” (p. 11). The conclusions should be ‘robust over time’.

The research has developed during 4 years of the researcher building knowledge. The researcher published any new knowledge so that it did not affect just the teaching done by the researcher but the teacher of the control group as well. The researcher made an effort not to change the overall way of teaching apart from the ways agreed by the department. Apart from the time of the introduction of the experimental lessons, teachers of both groups adopted a similar style of teaching. All students taking part in the research were from the same Sixth Form Centre. The teaching in the centre is based on the philosophy that students should be actively engaged in doing mathematics and physics rather than just be ‘taught’ by the teacher. The students follow a curriculum which is set by the exam board. They are all taught topics in a specific sequence guided by the text books used in the centre.

After establishing an effort made to keep invariance there is a need to establish variables and establish those that might have the significant effect on how students think and those which might hopefully not have affected the research too much.

The variables which are going to be considered are:

- *prior variables* which consist of factors dependent of the students’ background like they competencies, concept images already developed through previous experiences, their attitudes and cognitive preferences;

- *intervening variable*s which involve dynamic relationship in the classroom, social influences, students’ attitude to the tests and the interviews, students’ dedication to the course and teachers attitudes and teaching styles;

- *dependent variables* are how students’ can think flexibly, recognise the role of context and ability of assessing their own cognitive progress;

- *consequent variables* which are students’ success with the course and long-term changes in flexibility of thinking and conceptual understanding.

There will be other consequent variables which involve change in the researcher's style of teaching, the questioning techniques and even understanding of the topic but there are beyond the scope of this study.

5.2.3.1 Prior variables

The preliminary study and the pre-test helped to establish prior variables (students' existing concept images before undertaking the main study). The students had different teachers of Mathematics and Physics in the previous years and therefore different experiences, often dependent on the teachers' strengths. According to discussions with different teachers some of them made an effort to stretch students beyond the requirements of the syllabus, others took teaching vectors more seriously than others. Students were in the 'slowest' or 'fastest' groups which made the difference to the time the teacher could spend on the topic of vectors. All these variations have affected the way students learnt or assimilated their knowledge from the previous years teaching. This prior experience of students formed their cognitive units, concept images and schemas.

5.2.3.2 Intervening variables

In recent years the number of subjects students have had to study in year 12 has increased from 3 to 4 and they can drop one of these subjects in year 13. Some of them undertake mathematics only for one year to help them get to their chosen university. Students are now allowed to retake the exams as many times as they want, so they can enter year 12 with varied commitments to their study of mathematics, independent learning and abilities to reflect.

It is impossible to fully investigate if there are differences between teaching experiences two groups of students get in their classrooms. The control and the experimental group are also in three different physics groups but not split up in the same way as the maths groups due to other subject options they have decided to study in year 12. As the year 12 physics involves substantial teaching about vector quantities, students would have had varied experiences coming from that direction. There is also a small number of students who did not study physics (usually about 20%).

5.2.3.3 Dependent variables

This variable is tested through the post-test and the delayed post-test. It is hoped that it would depend on the students' experiences during the first two months of study. The students' from both groups were expected to make a substantial progress in their understanding of the concept of vector and of vector addition, especially through graphical representation as this is part of the Mechanics course during that period of time. Both groups post-test results were expected to be similar, however, due to the 'special treatment' of the experimental group, those students were expected to sustain the knowledge for a longer period of time. The students from the experimental group were given a facility to build a better conceptual base for the topic of vectors and so their results were expected to be better in the delayed post-test.

5.2.3.4 Consequent variables

This variable indicates students' future success when dealing with the vector problems, long-term changes in mathematical ability to think flexibly and be able to solve singular problems. This is tested by the delayed questionnaire, which shows the students' long-term ability to solve problems in vectors.

5.2.4 Qualitative and quantitative data collection instruments

The data collection instruments used in the main study include: pre- , post- and delayed tests; follow up interviews conducted after administrating pre- and post-questionnaires.; lesson observations; and interviews with the teachers.

5.2.4.1 Questionnaire

The questionnaire has been designed to give quantitative data. It was initially tested in the pilot study with the previous year of students. According to the students' responses, each answer was given the stage of the cognitive development separately in the graphical mode and in the symbolic mode, as described in figures 4.5 and 4.6 in chapter 4. Allocation of the stages was tested with other teachers.

5.2.4.2 Interviews

This qualitative data has been gathered with the help of the interviews with the staff before the course, interviews with the sample of the students conducted during the year, and lesson observation conducted during the time of the teaching experiment.

According to Cohen, Manion and Morrison (2000) in the interviews the greatest sources of bias are "the characteristics of the interviewer, the characteristics of the respondent, and the substantive content of the questions," (p. 121). During four years of this research, the interviewer kept practicing the interview techniques with many students. These interviews have been video-recorded and discussed with other researchers to enhance the reliability. The coding of responses has also been discussed with other researchers as well as with my research supervisors. Oppenheim (1992: 96-7) suggests several causes of bias in interviewing such as biased sampling, poor rapport, wording of the questions, poor prompting and biased probing, poor use and management of supporting materials, alterations to the sequence of questions, inconsistent coding of responses, selective or interpreted recording of data/transcripts and poor handling of difficult interviews. These were all considered when trying to improve the interviewing techniques.

The sampling has been limited by the students who refused to be interviewed (when letters were sent to parents to agree for their son or daughter to be interviewed, some refused for that to happen).

Poor prompting and biased probing has undergone improvement stages. The wording of the questions has been reviewed to limit the misunderstanding on the part of the respondent of what is being asked. There did not seem to be any problems with the rapport between interviewer and the interviewees.

This pre-course data collection (interviews with the teachers) served to check students' background. Different teachers have varied styles of teaching and use different resources. The interviews helped to establish teachers' expectations of students' knowledge of vectors. This prior variable has been considered in designing questionnaires and interview techniques. During the interviews the teachers were also consulted on the suitability of different questions in the tests and of the problems their students have encountered in Physics and Mathematics lessons where vectors were concerned.

5.2.4.3 Lesson observation

The students in the experimental group have been observed during the time of the teaching experiment. They were encouraged to comment while performing the embodied exercises in vector addition and when doing vector translations. The students who worked at the board were filmed as well as the group work and the plenary discussions. The control group was filmed during two of the lessons at the same time, to see if the students had any embodied experiences during their lessons.

5.3 Teaching experiment and plenaries

The objectives of the teaching experiment and the plenaries were to:

- help students in reappraisal of the roles of physical action and symbolic manipulation which relates actions having the same effect to mathematically equivalent concepts;
- encourage students to share ideas and to help them reassess and refine their knowledge;
- relate different aspects of mathematical theory to different aspects of mathematical applications in Physics.

The idea which was hoped to lead students from the action (embodied world) to the symbols with which they could operate (proceptual world) in their conceptual understanding of the free vector is presented in figure 5.1.

The experimental lessons were only conducted with the experimental group (group A).

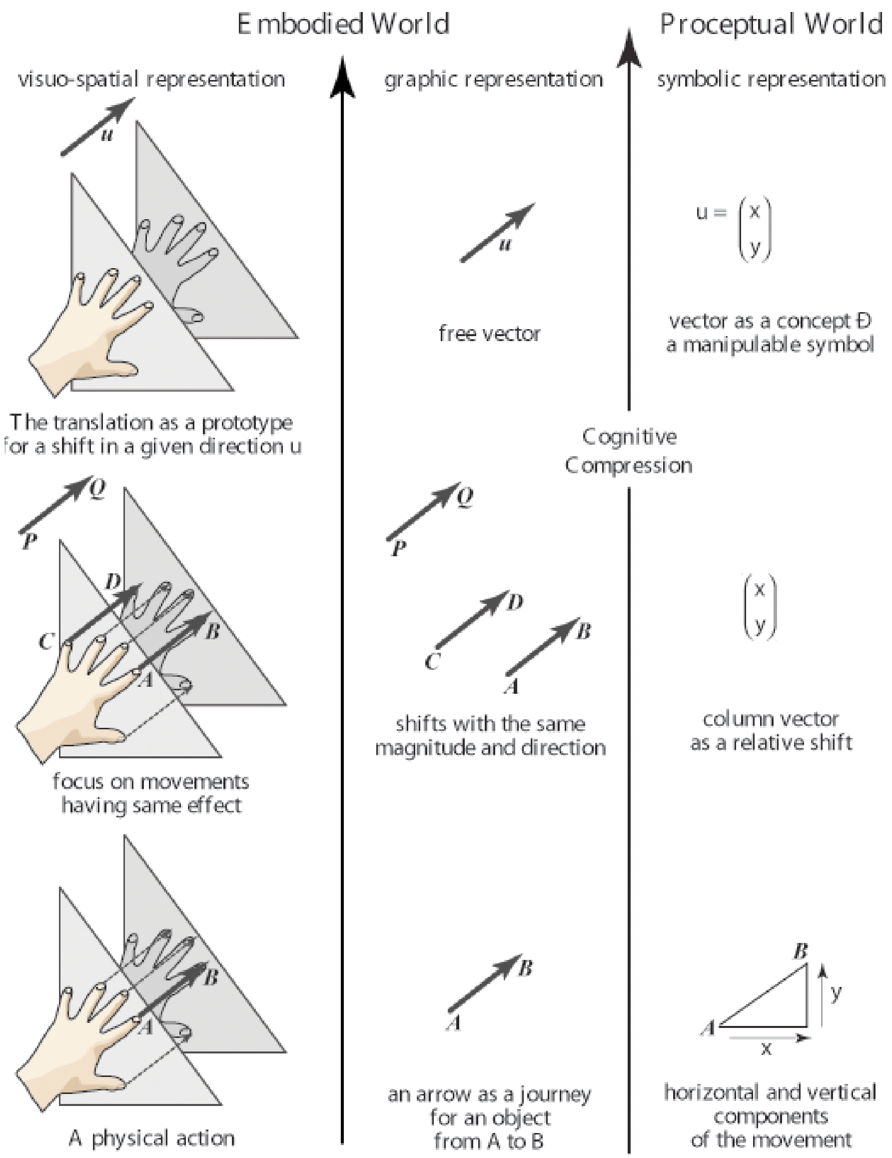

Fig. 5.1 Cognitive compression of the vector concept

First a volunteer student was asked to come to the white board, to put his hand on the board and draw around it. The student was then asked to move the hand across the board and draw around it again (movement from X position to A position, fig 5.2).

All students in group A were to discuss the way the translation could be presented. Some of the answers are shown in figure 5.2 (arrows 1, 2 and 3).

Fig. 5.2 Action of translation—experimental lesson

Another student came to the board put the hand in position A and translated it into position B (fig. 5.3).

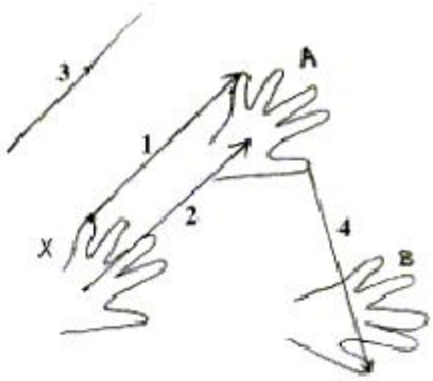

Fig. 5.3 Action with two translations—experimental lesson

The students were asked to discuss the way of representing the second translation on the drawing. The students agreed on the answer shown as the arrow 4 (fig. 5.3).

They were then asked to show the overall translation and agreed on the answer 5 shown in figure 5.4.

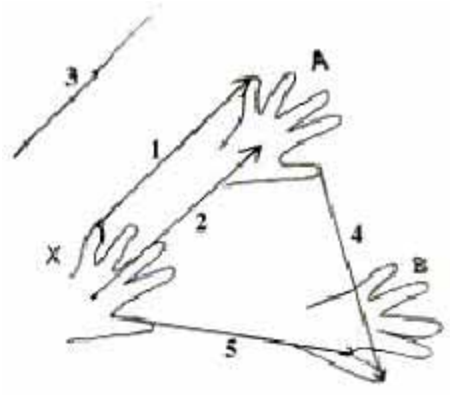

Fig. 5.4 Overall translation—experimental lesson.

The students were then invited to discuss how the answer can be shown mathematically. The preferred method to start was to calculate the horizontal and vertical movements (the way they are encouraged to do it in Physics) and add them together showing equal values for the vector 5. However being asked for more methods, some students came up with the idea of writing vectors in a symbolic form of a column vector and adding them together (fig 5.5), showing equivalence to vector 5. Eventually two students came with the idea of adding vectors graphically. They were encouraged first to work on the idea in small groups and then as the class discussion. Although students all knew the rule of moving vectors 'nose to tail' in order to add them, this was not easily remembered in the above situation.

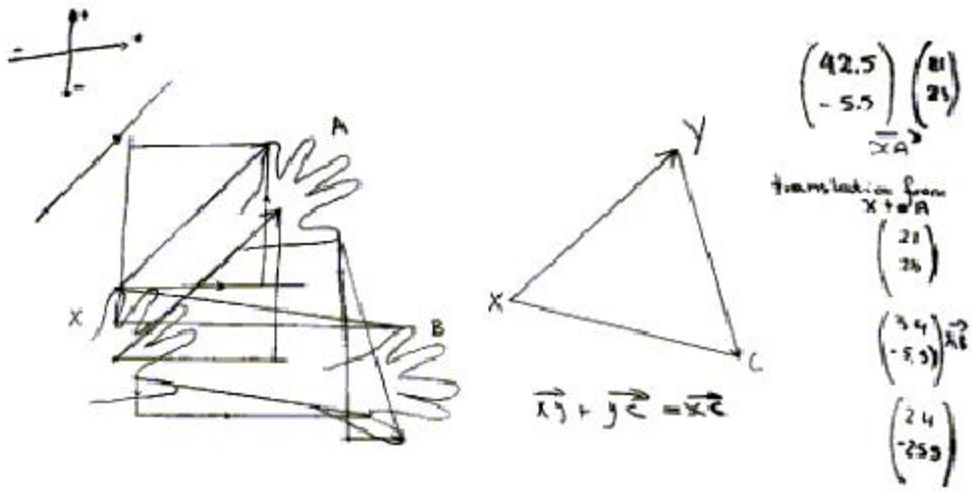

Fig. 5.5 Action of addition of two translations—experimental lesson.

The students were asked during the next lesson to repeat the exercise but with a triangular shape which they pushed with their hand, during which they had another type of embodied experience.

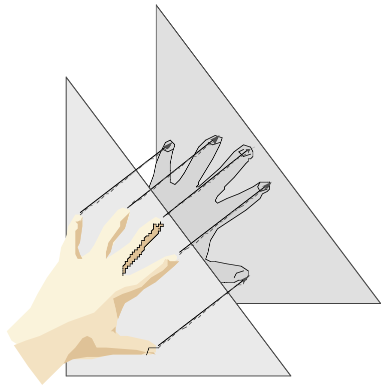

Fig. 5.6 Moving object—experimental lesson.

The students were asked first to draw a shape and cut it out, one of the students was asked to put his shape on the board and translate it (movement without rotation, fig. 5.6). Then the exercise followed the same steps as in the previous experimental lesson, but with the triangle. Not all the students were able to recall experiences gained from the previous lesson and there was still a lot of discussion in plenaries of the geometrical way of vector addition.

During the third lesson the students were asked to add forces acting on the objects, showing the addition on the paper with the precision enhanced by using a ruler and a set square to give them an embodied sense of the operation. They had to move the set square along the ruler with their hands to draw precise parallel lines and then measure the magnitude and repeat it with the precision on the translated vector.

5.4 Summary

The research methodology and the rationale for various ways of collecting data and for the way the teaching experiment has been conducted were described in this chapter. The data collection consisted of three major components: questionnaires, interviews and observation of students during the teaching experiment. Additional data has been conducted through research of the textbooks the students use, interviews with teachers and collecting students' results from external exams and internal tests. These additional components also provided means of triangulation and in establishing the prior variables. Changes to those variables were documented using pre-test, post-test, delayed post-test and interviews. The quantitative and qualitative components of the main study were designed to address the three main research interests:

1. discover what makes some students link the real world activities with mathematical symbolism in a meaningful way;
2. discover what prevents some students in making these links;
3. to increase students understanding of these links through specific teaching techniques of which the main part are the plenary sessions and make them more flexible in their thinking.

Pre-, post- and delayed post- tests were designed to examine the above points and to provide data showing if the teaching experiment had any effect on students learning and their flexibility of thought. The class observation and interviews were conducted in order to add a qualitative component to the first two points of interest and provide the insight into reasons for students' difficulties and strengths.