Chapter 8

Main Study: Qualitative Data Analysis

Interviews with the teachers

8.1 Introduction

This chapter focuses on the qualitative issues through individual interviews with teachers.

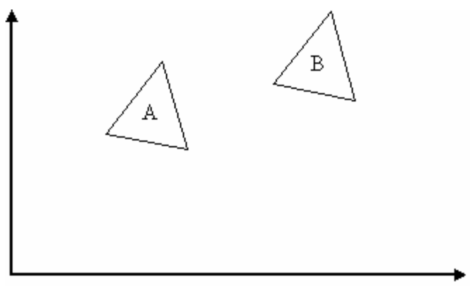
The interviews with one Physics teacher and two Mathematics teachers served the purpose of finding differences in the teaching and expectations of teachers in Physics and Mathematics. They were conducted during the period of the Main Study and were based on the test questions.

The three teachers were shown the questions and asked what they thought the students' responses would be. The Physics teacher was coded as P and the Mathematics teachers as M1 and M2. All three teachers are females.

8.2 Interview with the teachers

The selection of the test questions are shown in figures 8.1 to 8.5 and a sample of the typical teachers' responses are quoted below each figure.

1)In the picture the triangle has been translated from position A to position B as shown below:



(a) How can you represent the translation of the triangle? Show it on the picture

(b) Can you draw a vector starting at the origin (0,0) which will represent the translation of the triangle from A to B? If so, show it on the drawing

(c) Can you draw a vector not starting at the origin and not touching either of the triangles which will represent the translation from A to If so show it on the drawing

Fig. 8.1 Test question 1: Represent translation

P: "They will choose a specific point for (a). [...] in (b) they should be alright, but they will be confused in (c). [...] Sometimes they ignore the direction and don't place an arrow on the line."

M1: "I would expect them to go across and along. To go from a point on shape A to a corresponding point on B is another building block. [...] I am not sure if at the beginning they connect both together."

M2: "Many will be happy with representation of the translation. Only a few will think of the horizontal and vertical."

Summary:

The Physics teacher seems to think that the students would have no problem with an equivalent vector starting at the origin, but would have a problem with a vector starting elsewhere off the triangle. She did not mention horizontal and vertical components, in contrast to both Mathematics teachers. Teacher M1 comments that from using the horizontal and vertical components to considering a vector from point on A to a point on B is "another building block", whereas teacher M2 thought they will be "happy" with the translation and only "a few" would use horizontal and vertical components.

Comment:

At first, the Physics teacher's response seemed unusual to me, however, if one views the instruction as asking students to move a triangle in a physical way, this cannot be done unless one places one's hand on the triangle itself. Hence the physics teacher's response in part (c) may be based on real world activity, whereas in part (b) it is based on the familiar task of drawing a position vector from the origin in a manner the students will have met in class. In this rather subtle way, 'real world' experiences might therefore interfere with the mathematical notion free vector. She also expects some students to "ignore direction" and therefore not realise what the vector represents. She implies that they do not realise the meaning of the graphical symbol of vector. From the preliminary interviews with the physics teachers it seems that they use the numerical methods to calculate resultant vectors and yet the idea of representing the translation with the horizontal and vertical components came from the teachers of mathematics. One of the Mathematics teachers showed awareness that using an arrow instead of just horizontal and vertical components is another stage in the cognitive development: "To go from a point on shape A to a corresponding point on B is another building block". So the teachers expressed there might be problem with the symbol of an arrow, flexibility of using that symbol flexibly as a mental concept and although they expressed that even if students know the equivalence of vectors, they might not have compressed that knowledge into the concept of free vector.

The next set of questions that the teachers were asked to comment upon (figure 8.2), involved addition of two vectors, where each example is 'singular' in some way (see section 6.1.2)

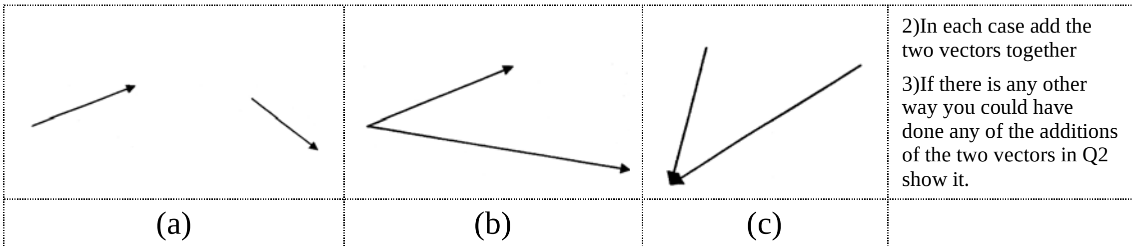


Fig. 8.2 Test questions 2 and 3: Add two vectors

P: "This is confusing, especially part (a). They might think that two vectors should be attached to each other. [...] In (b) they will add them using the parallelogram. [...] They are taught to use the parallelogram if the two vectors are connected at one point. When you give them a question with the two tugs, they find it more useful to answer using a parallelogram. They can see that it is pulled in a specific direction." (Interviewer: "What about the triangle?") "The triangle is used for resolving.[...] I would use the triangle with them if it was a displacement." (Interviewer: "What about part (c)?") "Confusing, they do not get the idea that the arrows have to follow ends. Some might resolve using matrices." (The teacher refers to the use of column vectors). (Interviewer: "What about question in part (c)?"). "They might do it horizontally and vertically."

M1: "They will do them by placing 'nose to tail' or will use column vectors. For some, these vectors might seem fixed in space and they will draw another two vectors between them to close the polygon. In part (b) they might think that they are connected in the wrong way and simply join them up with a third vector. Later on in the year they should use a triangle or maybe a parallelogram. They would think of the translation as a displacement. Parts (b) and (c) are two different visual images. They should be able to answer part (c) by the end of the course. They might use a triangle rather then a parallelogram. [...] Part three should make them think, and maybe even amend their answers to question two."

M2: "They might be able to answer part (a) if they have the idea of moving them 'nose to tail'. In (b) they have to disrupt a diagram, so only some might do the parallel displacement of the bottom vector and use the triangle rule. I don't think that they will think of a parallelogram. In (c) if they answer they will definitely think of translating a vector. [...] Question three should make them think that there are different ways of doing things. If they already used a triangle rule one way they might use it the other way. [...] They might put vectors 'nose to tail' without drawing the resultant because sometimes they are taught to do this and expect them to fill the gap, not realising they will not know it should be filled."

Summary:

The Physics teacher refers to real-life situations that occur in teaching, when for example 2 forces ("the two tugs") are acting on an object, or two journeys that follow each other. The teacher thought that the students would use the parallelogram law of addition in case (b). She thought that the students would be confused about part (a) as it has no real-life significance. This suggests that, from a physical point of view, part (a) is a 'singular' case. She thought that students would find part (c) an unusual situation and would resort to numerical methods such as column vectors (horizontal and vertical components). She was aware that some students have difficulties moving vectors 'nose to tail'.

The Mathematics teachers thought that the students would answer differently before the course and after the course and allowed therefore for a conceptual development. They both realised that if students have the idea of using the 'nose to tail' technique of adding vectors then they should be able to solve all three parts. However one teacher (M1) thought that students might think of the vectors as 'fixed in space' and therefore have a problem with the questions unless they use the numerical method. The other teacher (M2) expressed the idea of vectors being 'fixed in space' as diagrams which the students might not want to "disturb". The first teacher thought that the students might use the triangle law of addition in part (b) or maybe a parallelogram law, but the second teacher thought that it is unlikely that students will use the parallelogram. They realised that parts (b) and (c) present "two different visual images" and that part (c) being 'more difficult' should be solvable to the students by the end of the course. One teacher mentions that the students might place vectors 'nose to tail' without showing the resultant. The second teacher mentions that the students who are familiar with the technique of 'nose to tail' but not addition, might instead feel that the addition means placing vectors next to each other (like a journey).

Comment:

The Physics teacher thought about these questions in a physical way: "two tugs", "use of the triangle if it was a displacement." She also indicated that the parallelogram rule is for forces and the triangle rule for displacement. It might be that Physics teachers do not use the 'nose to tail' technique when adding vectors. She also treated question (a) where vectors are separated as a 'singular' case. It was neither a journey nor forces. She was not considering vectors in a general mathematical context, focusing only on what the diagram could mean physically. When we consider that students learn vectors in physics first, then it would seem realistic to consider that it would be very difficult for them to conceptualise a vector (arrow) as a mathematical symbol without referring in their mind to some physical situation. This is the reason why the experimental lessons were conducted by starting students working on a physical object in a flexible physical context to allow them to realise the mathematical implications of a free vector.

The Mathematics teachers mentioned two ways of solving the questions. Implicitly, according to the teachers, some students have knowledge of moving vectors 'nose to tail' and others see them as fixed in space and will either do the addition of components numerically or use some other method based on partial knowledge. The mathematics teachers used the meaning of the parallelogram and triangular laws of addition interchangeably supported by phrases such as "translating a vector", "parallel displacement" and "close the polygon." In their language they considered the questions partly as set in the mathematical context, but with hints of a physical context by saying "translation as a displacement" and "fixed in space".

All 3 teachers saw that the arrows coming to one point in part (c) represent a 'singular case' (as something that students would have not met before).

The teachers also realised that some questions are 'harder' which the researcher considered to be at the higher conceptual level. In particular, teacher M2 used the term "disrupt" to represent the movement of vectors in (b), which indicates that she regarded it as being significantly different from the case students usually meet.

This use of the word "disrupt", which the teacher M2 mentions on several occasions interests me very much, as it seems to give the diagram a physical meaning. It is as if the teacher herself embodies the diagram with a physical meaning. Given the many subtle ways in which physical meaning interfere with the mathematical meanings, I found the phenomenon very significant.

The overall impression from the teachers was that some questions are 'singular' as students would not have met such situations before. The physical implication of the question could have affected the way it is responded to by students. In Watson, Spyrou and Tall (2002) we started by regarding questions (a) and (c) as more suitable for use of the triangle law of addition, while question (b) as more suitable for the parallelogram law. The teachers also mention both laws in some of their responses. However when we triangulate this which the students' test responses only one student used the parallelogram law of addition.

The next question, (Figure 8.3) asked the students to add 3 free vectors. The teachers' responses to the possible answers that students might give to the question shown in are listed below.

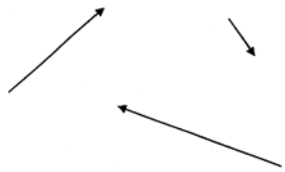


Fig. 8.3 Test question 4: Add three vectors

P: "We never do anything like this, so I am not sure."

M1: "I think that they will be tempted to join them together. The problem is that they need mathematical equipment to do this and they are not used to using it. Pupils had to do Technical Drawing years ago but these skills are not required any more. They might have a problem connecting them if they happen to cross each other. It might seem wrong."

M2: "They will choose the longest and then join them in the order of size. They probably will do it below, so they do not cross."

Summary:

The Physics teacher said that the students had no experience with this type of question and therefore their responses will be unknown to her. The Mathematics teachers thought that the students might "join them together" meaning place them 'nose to tail'. However one teacher was concerned about the students' lack of skills for such an activity, due to lack of technical drawing skills; the other was concerned about students' confusion if the vectors cross.

Comment:

The Physics teacher indicated that the context matters and if the question cannot be used in the physical context it is not valid from her subject point of view. Actually the question could be placed in the physical context which was used in the experimental lesson. It could have been for example three displacements each taken from a different point on the object, or three forces placed at three different points on one object, but since in her subject such questions are not being considered this did not come into consideration. It can be concluded that in Physics the students are not taught the idea of the free vector which can be used in any required context.

One of the Mathematics teachers (M1) implied that the question would be difficult to answer without the use of mathematical equipment, and that students are no longer taught drawing skills. The other teacher (M2) was concerned about the difficulty that might occur when the drawing leads to two vectors crossing. This is consistent with the idea that such a case is a ‘singular’ and not generic because it contains specific properties that need not occur in the general case. (Such a case happens again in a later question discussed with the teachers (in figure 8.6c). Both teachers seem to concentrate concentrated on how students can use the procedure of adding vectors and what might prevent them using it.

The next 2 questions the teachers commented on are shown in figure 8.4. The questions were placed in the specific physical situation and restricted the number of vectors students were supposed to use, but otherwise were open-ended.

Draw a representation of three forces and add them together.	Draw a representation of two displacements and add them together.
(a)	(b)

Fig. 8.4 Questions set in two different contexts

P: “In the first one they would draw forces acting at one point, all in line, at least to start with, and later they might draw an object and draw forces acting vertically and horizontally. In the second one they will draw one journey followed by the other.”

M1: “After the course they should draw forces acting on a particle, but then it is not easy to add. [...] Bit like question 2(a) [Figure 8.2b]. For displacements they would draw one after the other. You cannot displace from the same point twice.”

M2: “They would draw them from one point and then they would have to disturb the diagram to add them. With forces they would think of something acting on a particle. I think in a question with displacement they would have to draw a shape to displace. Most students would have difficulty to do this question without something to refer to.”

Summary:

All three teachers agreed that the students would use three forces acting at one point in part (a). The discrepancy occurred in their anticipation of the way the students would add the forces together. The Physics teacher gave a simple solution of drawing (the components of) the forces only horizontally and vertically.

All three teachers distinguished between the two different contexts and anticipate that students would show the difference between the two ways of representation. One Mathematics teacher (M2) suggested that students will find difficulty in representing the displacement without the object to act on: “in a question with displacement they would have to draw a shape to displace” and also that students might find difficulty adding vectors which are connected in the question, again using the expression “they would have to *disturb* the diagram”.

Comment:

Since a lot of mathematics has been cut out from the Physics syllabus at all stages of studying of this subject, a solution showing forces as vectors in 2-Dimensions only, as anticipated by the Physics teacher, would have been the most likely response by the students. However she thought that the concept of journey will be stronger in terms of vectors following each other. The mathematics teacher M1 agreed about the concept of journey with the Physics teacher, however teacher M2 was concerned that students need an object to act on. The teacher M1, by saying "you cannot displace from the same point twice", intimated that if a displacement is measured from the origin, this is saying that a *second* displacement must be drawn from where the first ends, you cannot go back and displace the first point twice.

Both Mathematics teachers agreed with the assumption of the Physics teacher about drawing forces acting on an object and therefore being drawn from one point. However neither of them realised the way students are taught to add forces in Physics and thought and therefore anticipated problems with addition. They seemed to know instinctively that students have problems with dealing with 'free vectors' geometrically in 'singular' type of cases. Thus M1 said what the student would try to do, which in the context of forces means drawing them from a point, (which may give them difficulties) but for the second part (displacements) she said that they would place them one after the other. The teacher M2 expressed the opinion that students need an object to perform an action on, which means she anticipated a very physical attitude to addition of displacements.

All three teachers suggested different approaches from the students according to the context they would operate in. One teacher suggested that in the physical context students would only be able to follow the procedures if they first had an object to act on, and therefore implied that students operate differently with vectors in different physical contexts.

The last two questions (figures 8.5 and 8.6) are very different from the previous questions.

Using the drawing below, or otherwise, add:

(a) $\overrightarrow{AB} + \overrightarrow{BD}$

(b) $\overrightarrow{DA} + \overrightarrow{ED}$

(c) $\overrightarrow{AB} + \overrightarrow{AE}$

(d) $\overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{DC}$

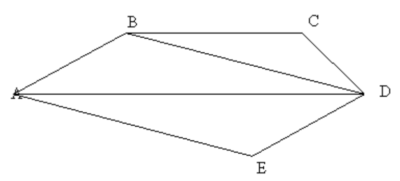


Fig. 8.5 Question on vector addition set in the diagram

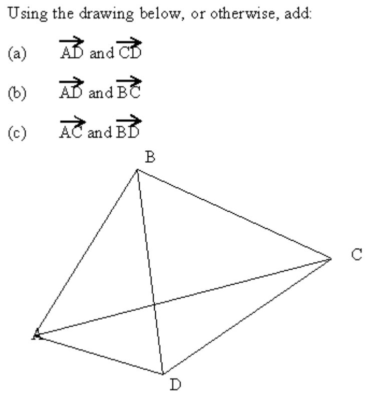


Fig. 8.6 Singular question on vector addition set in the diagram

The students had to pick their information from ‘busy’ diagrams and recognise the specific symbolism for the displacement vector (starting at a specific point and ending at another specific point). The questions in figure 8.5 should be familiar to students because it is of a type encountered in the previous year’s course. However the questions in figure 8.6 would not be familiar because they require the students to draw additional lines not in the figure. The teachers’ responses are shown below.

P: “We never do anything like this, so I do not know” (Interviewer: “Any thoughts”) “Some of these [points to questions shown in figure 8.6] are impossible [...] the answer does not fit on the drawing.”

(Interviewer: ”They do not need to be a part of the drawing”). “It is not clear, I think the students would assume that it should be part of the drawing.”

M1: “Most students should do the first one (Figure 8.5). This is a GCSE material (Year 11). Part (b) seems more difficult but the students who want to do ‘A’ levels are used to keep going so it does not matter if the most difficult question is in the middle. [...] The second one (Figure 8.6) is very difficult. It is a mental jump to draw a new line. They would expect the answer to be in a diagram. This might go beyond their understanding. It would require a confidence to answer. [...] Only those who understand about vectors will be able to do this question.”

M2: "Looking at the first one (Figure 8.5) they will put arrows on the lines when trying to add the vectors. The arrows would help them to think if they are going the right way. [...] Students would assume that *ABDE* is a parallelogram and use the opposite side in part (b) to make the second vector follow the first. It is back to a journey. They should be able to answer these questions. The second one (Figure 8.6) is much more difficult. [...] They might feel uncomfortable moving out of the drawing and even when drawing outside they might try to compare it to the sides already present in the diagram."

Summary:

The lack of context mattered to the Physics teacher. She did not want to commit herself to speculating on students' responses in an area which was unfamiliar to her. However, when asked for some sort of comment, she responded that the second set of questions is more difficult as students have to draw the additional lines as their responses. She therefore recognised that the questions presented in figure 8.6 are 'harder'.

The Mathematics teachers thought that only part (b) of the question presented in figure 8.5 could cause a problem. The question asks students to add vectors which are written in a reverse order than the way they follow each other. They also anticipated students would have a problem when answering the set of questions presented in figure 8.6.

Comment:

All three teachers recognised that the question presented in figure 8.6 could cause students problems and in fact this was a 'singular question' as the students never met anything like this before.

The Mathematics teachers also recognised that part (b) of the questions presented in figure 8.5 is different. In this part students could have used the commutative law of addition where $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$. The teachers however did not consider students using that law and thought that they might use the rule of the equivalent vectors. The teacher, M1, implied that dealing with free vectors is another stage in the cognitive development: "It is a mental jump to draw a new line".

8.3 General Summary

The main hypothesis in chapter 7 states that:

> Teachers can help students develop the notion of a translation as a free vector through focusing on the effects of physical actions, linking graphic and symbolic representations, so that the concept of free vector is constructed as a cognitive unit that may be used in a versatile way in a range of different contexts.

If we triangulate the teachers' interviews and the students' responses to the tests, we may conclude that teachers are very aware of most mistakes that students might make.

For instance, one of the Mathematics teachers comments, "They might put vectors 'nose to tail' without drawing the resultant, because sometimes they are taught to do this." From the results of the quantitative tests, it is clear that a number of students added two vectors $\overrightarrow{AB}+\overrightarrow{BC}$ by simply drawing them 'nose to tail' but did not draw the resultant $\overrightarrow{AC}$. The theoretical framework would suggest that the student may see the addition of two vectors as a journey from *A* to *C* via *B* and not as the higher level concept of free vector. However, this was not a concern expressed by any teacher. It may be that in coping with a crowded syllabus, the quest is often to get the students to respond correctly rather than seek for subtle reasons why they may make mistakes.

If students have no experience of the idea of the free vector in placing vectors 'nose to tail' then they are less likely to make the connection of an arrow being a symbol for a free vector that can be used the same way regardless of the context.

The Science teacher said, "... sometimes they ignore the direction and not place an arrow on the line", which means that she is used to students misinterpreting a graphical symbol of a vector which could cause them difficulties in coping with the concept of free vector in any context that does not have a physical meaning. She did not expect students to make connections between the actions and symbols.

The second hypothesis in chapter 7 states that:

> Students who were helped in building a concept of a free vector can add vectors in 'singular' cases, not just generic ones; they can also use free vectors independent of the context the addition is set in and realise that the commutative law applies to the vector addition.

If we again triangulate the students' test responses with the teachers' comments we can see that the teachers were aware that some questions are more difficult than the others (singular cases) but the views from the Physics teacher and the Mathematics teachers differed substantially.

The Physics teacher expected students to have problems with questions where vectors were drawn separately (figures 8.2 (a) and 8.3). She did not expect the student to use the idea of the free vector at all and every time the question could not be connected to the *obvious* physical context, she treated it as a 'singular' case.

Generally the Mathematics teachers expected students to use the idea of 'free vectors' at the end of year 12, but did not realise that the concept was essentially required by the end of their year 11 teaching; the text book introduces the concept of vector (through the implied APOS theory) by moving from action on objects through equivalent vectors to the concept of free vector. However they spoke in a way which indicated a subtle sense of the steps in the development: For instance, one teacher said, "I would expect them to go across and along. To go from a point on shape A to a corresponding point on B is another building block". This suggests an awareness of the conceptual change that is required to move from one stage to the next. When the need of drawing the equivalent vector arose, one teacher said: "It is a mental jump to draw a new line." In the case of the 'singular' question in figure 8.2 (c) one of the teachers responded: "This might go beyond their understanding" and the other teacher said: "They might feel uncomfortable."

These comments all show the sensitive realisation by the teachers of the possible areas were students may have difficulty. It is matched by the difficulties that are experienced by many students prior to the course and by some of the control students after the course. The success of the experimental group suggests that it may be a help to other teachers to be aware of the simple idea of ‘focusing on the effect of an action’ which may help students form the idea of free vector in a more versatile and confident way.