Chapter 9
Main Study: Qualitative Data Analysis
Interviews with the students

9.1 Introduction

This chapter focuses on the qualitative issues through individual interviews with the students. The interviews were intended to gain a greater insight into:

- students' use and flexibility of language when discussing problems connected with vector addition;
- students' focus of attention at any given time (whether it is on actions, or procedures or on the effects of those actions and procedures);
- the way in which different contexts affect their thinking;
- their flexibility in dealing with different modes of operation (graphical/symbolic).

The interviews were also intended to test if the students were placed, on the basis of the test analysis, in the right categories according to the theoretical framework developed in chapter 3. It was intended that interviewed students should be selected from a spread of different categories (uni-modal, higher uni-modal, multi-skilled, versatile & fully integrated) as well as from both groups (experimental and control).

There were two sets of interviews: after the pre-test and after the post-test. Different students were interviewed each time. The extracts from the interviews with students are presented in the two sections below (9.1, 9.2).

9.2 The interviews following the pre-test

The pre-test interviews were conducted with four student coded as:

S1: answered in an ‘*intuitive*’ way;

S2: answered mainly *symbolically* and has been classified as *higher uni-modal*;

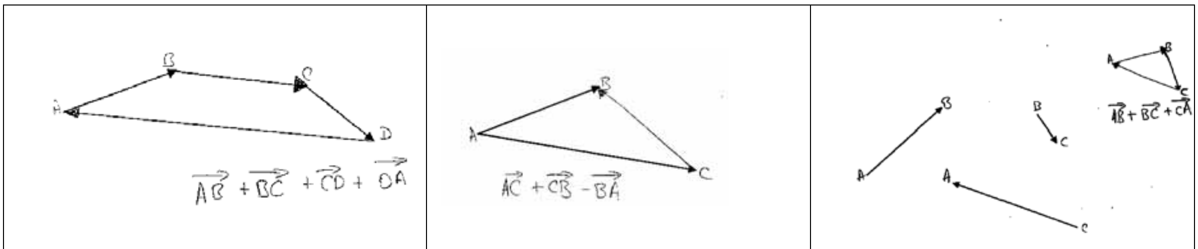
S3: answered mainly *graphically* and has been classified as *higher uni-modal*;

S4: classified as belonging to the highest *fully integrated* category.

The students S1 and S2 were from the experimental group (A) and students S3 and S4 were from the control group (B).

9.2.1 Student S1

The examples of the student’s S1 responses are shown in figure 9.1.



(a) (b) (c)

Fig. 9.1 Student S1: examples of responses to the pre-test

Figure 9.1 parts (a) and (b) are the student’s pre-test (T1) responses to the questions shown in chapter 6, figure 6.2 (a) and (b), while 9.1 part (c) is the response to the questions asking for the addition of the three given vectors (figure 6.4). In the first question (figure 9.1 (a)) the student was asked to add two vectors, which he named $\overrightarrow{AB}$ and $\overrightarrow{CD}$. The student filled gaps between vectors with extra vectors ($\overrightarrow{BC}$ and $\overrightarrow{DA}$). In the second question (figure 9.1 (b)) the student was asked to add two vectors, which he named $\overrightarrow{AB}$ and $\overrightarrow{AC}$. The vector $\overrightarrow{CB}$ was drawn to close the gap. In the third question (figure 9.1 (c)) the student was given three vectors to add, which he named $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$. He ignored the magnitude of the vectors in this question and just considered the approximate directions to form a triangle. This is why the student was categorised as ***Physical Intuitive.*** Parts of the interview are presented below (I – stands for the interviewer).

I: Look at your answers in the first part of question 2 can you explain it to me? (fig. 9.1a).

S1: In this I drew them clockwise.

I: What is the result of the addition?

S1: Sum.

I: Which one is your sum?

S1: A there to D [points].

I: So what would this one mean? [The interviewer points to BC.]

S1: This one just joins these two vectors so they can be added together.

I: So what were you looking for?

S1: Continuity.

I: What about the next part? (fig. 9.1b).

S1: I was trying to do them the same way?

I: How did you do them the same way?

S1: These two (points to B and C) go in separate directions.

I: Did it worry you in any way?

S1: No

I: Can we look at your answer to Q4? (fig. 9.1c). How did you add the vectors?

S1: I connected all the vectors together so it will be easier to add them all together.

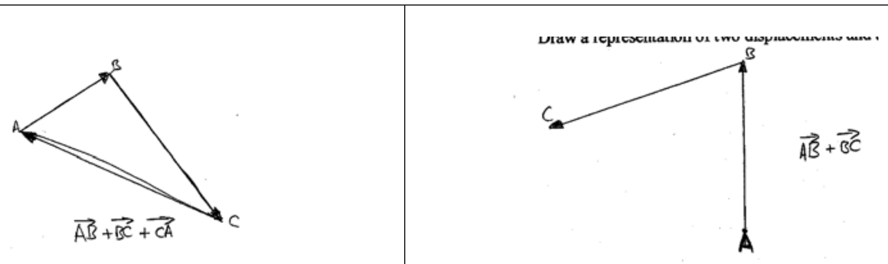
I: Didn't worry you that they have different length on your drawing

S1: I did not draw them to scale but to different scale just to give a general idea of how to add them.

I: Didn't you think of doing some measurements when adding the three vectors?

S1: No

The responses the student gave to the questions in the two different contexts are shown in figure 9.2.



(a) (b)

Fig. 9.2 Student S1: examples of responses to the pre-test

I: When you were asked to add three forces together, you drew a triangle [fig. 9.2a] but when you were adding two displacements you did not [fig. 9.2b], can you explain it?

S1: I think I forgot a line joining beginning to the end (point from A to C, figure 9.2 (b)).

I: What would this line represent?

S1: It would be a displacement from the first point to the last point (points from A to B)

I: Can you explain it a bit more?

S1: Distance, instead of going from A to B and B to C you can go shorter distance from A to C.

I: In the other question, if all three forces form a triangle does it mean anything physically?

S1: Not clearly, no.

I: Could there be a situation when they do not meet like that, when there is a gap left?

S1: Didn't come back to the starting point? What would it mean if they didn't?

I: Would it make any difference if they did not meet?

S1: Then you would not be able to add them all together.

I: Can you draw for me an example of forces acting on an object?

S1: If you have a particle, you would have the gravity, the resistance and if you were pushing it from one side you would have a force acting this way [student draws two vertical forces and one horizontal].

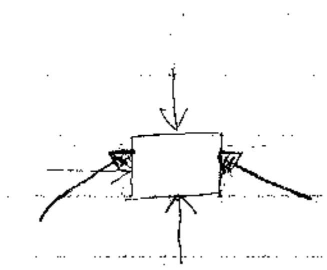


Fig. 9.3 Student S1: example of response to the pre-test

I: Can you add them for me

S1: You add the y components, you would multiply the gravity by the mass and this has to be equal to the force up.

I: Why do they have to be the same?

S1: Because if it remains still in the horizontal angle then it is not going downwards or going upwards

I: If you had two additional forces on your object (interviewer draws two forces) and the object would not be moving, how could it work?

S1: You could work it up by using a force and then using an angle and use cosine to work out what this component is and what this component is and work it out (student waves his pen in the horizontal direction below each of the forces, but without the particular direction).

I: Could you do it graphically?

S1: Well, [student starts by drawing horizontal components and vertical components].

Summary

The student S1 does not show the flexibility of language when discussing the problem connected with vectors. Although he knows what it means to join vectors 'nose to tail' he has no awareness of the notion of equivalent vectors and vector addition. He does not fully understand the symbolic representation in a graphical sense. The student uses the word "continuity" to mean that one vector follows immediately after another, without a gap. He attached labels to the graphical symbols of vectors (the way it is taught in the Year 11 text book) and shows addition in that way but without showing the result — the *effect* of that addition.

When adding vectors, the student focuses on the idea of continuity in two ways: either by adding extra vectors to fill in the spaces, or joining them together one after another, but without showing the resultant. He used a ruler to draw all his answers except the one shown in figure 9.10 part (c) as this could possibly defy his own theory about vector addition. He confirms it by saying that if the vectors did not join then "Then you would not be able to add them all together." He does not connect the physical effect of the addition in a more general way, as a total effect. Although he draws the arrows in figures 9.1 (a) and 9.2 (a), when he describes what is happening he indicates the correct direction and he seems to have some concept of ***the same effect*** in the embodied sense of a journey: "Distance, instead of going from *A* to *B* and *B* to *C* you can go the shorter distance from *A* to *C*".

When he adds vectors, in any context he looks for the continuity. He can operate in one directional environment as far as forces are concerned. He indicates it by saying that, for the object not to move, the vectors should be of the same magnitude but in the opposite directions (his explanation to his own drawing in figure 9.3). He also thinks that the forces should close the loop. When asked what would happen if after putting forces graphically together there was a gap left, he seems dismayed and answers: "Didn't come back to the starting point? What would it mean if they didn't? [...] Then you would not be able to add them all together." His preferable mode of operation when he thinks of forces, under different angles than vertical, seems to be symbolic. He suggested adding the horizontal and vertical components for the vectors drawn (in figure 9.3) under different angles. However in the context of displacement he thinks of a journey which follows the vectors as they are placed one after another, in physical terms. He does not consider the addition of two displacements giving the total effect but simply as getting from the first point to the last one in a shorter distance.

From the above discussion with him it can be concluded that he is partly in a ***physical intuitive*** class and partly at stage 1 of the graphical mode and stage 2 of the symbolic mode. So maybe he could be classified as ***uni-modal*** however the dividing line is not clear.

Comment

The student's understanding seems to be at a very low stage in the graphical mode: putting vectors 'nose to tail' to give continuity, as if it is a journey of which one part has to start where the previous finishes and working with forces in one direction. However he has no awareness of the idea of equivalent vectors or free vectors. This student seems to be graphically locked in either a context of journey or forces but only in one direction in dealing with vectors. He might have answered better in the symbolic mode if everything was set on grids. However, he sensed from the type of questions asked that the graphical method was the preferable one but he was not confident with it.

9.2.2 Student S2

The second student (S2) responded, in the pre-test, mainly using the symbolic mode. The examples of her pre-test responses are presented in figure 9.4. The relevant part of the interview is shown below the figure 9.4.

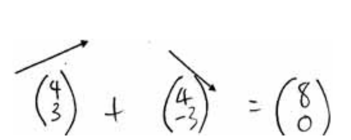
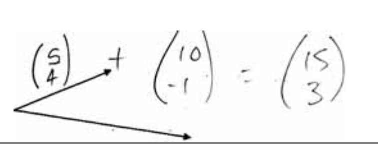
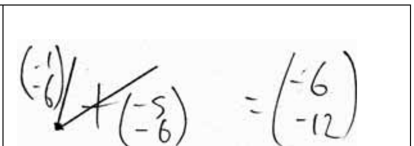
		
(a)	(b)	(c)

Fig. 9.4 Student S2: examples of responses to the pre-test

I: What do you think the question is asking you to do?

S2: I thought you want an actual number.

I: Why do you think I wanted a number?

S2: Because of the way I was taught, we were taught to put them on the grids.

I: So what is your technique in a graphical mode?

S2: I put them nose to tail [places them 'nose to tail without any precision, figure 9.5].

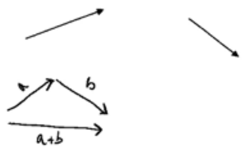


Fig. 9.5 Student S2: response to the interview question

I: What about the last question?

S2: [Student connects them again nose to tail on a separate drawing, using a ruler, figure 9.6].

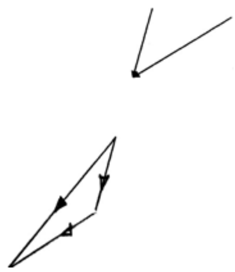


Fig. 9.6 Student S2: the interview response to the singular question

I: The next question asks to do it in a different way. How would you do that?

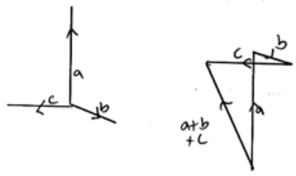
S2: You can work out all the vertical and horizontal stuff, I am not sure.

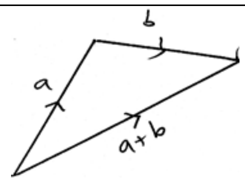
I: The numbers you have used in your answer, what meaning did they have for your resultant?

S2: I think I just used centimetres.

I: Why centimetres?

S2: I did not think it really mattered what the scale was.





(a) pre-test response	(b) interview response

Fig. 9.7 Student S2: responses to the different contexts pre-test questions

I: What about the next question? (referring to the question set in the context of forces; the test response in figure 9.7, part a).

S2: I am used to forces going out.

I: What about the next question, you left it blank?

S2: I guess I just didn't really know what you meant by displacement. Are displacements the same as vectors?

I: So doesn't Physics talk about displacement?

S2: Oh, it is like movement? Isn't it? in a certain direction, but isn't it what a vector is? [...] So would you like me to do just the same thing again?

I: Would you like to answer it now?

S2: [student draws two vectors following each other and the resultant as shown in figure 9.7 (b)].

I: When you started answering you first thought was to use numbers. Why do you think that the number answer was your first choice?

S2: Because if you just draw there are no numbers involved and you should have some numbers in the answer. Drawing the picture doesn't really answer the question.

Summary

The language the student S2 uses lacks flexibility when different contexts are concerned. She is not sure about the concept of vector in case of displacements but seem to be quite confident with the idea of vector to represent forces.

She is able to focus on the effect of the procedures which she shows when adding vectors even in the singular cases (figure 9.6) as well as in case of adding forces (figure 9.7, part a). When she realises that displacement is "movement" and therefore can be represented by a vector she also can use addition in the second physical context.

She proved in the interview to have flexibility in dealing with different modes of operation, which was not so clear from the test, as she answered only one question graphically. She explained that she used numbers because this is what she though was expected in vector questions. She is not sure about the scale as only met problems presented on grids till now. She was also under impression that "drawing the picture doesn't really answer the question".

It would seem that the student S2 could belong in the '*versatile*' category and not the *higher multi-skilled category*.

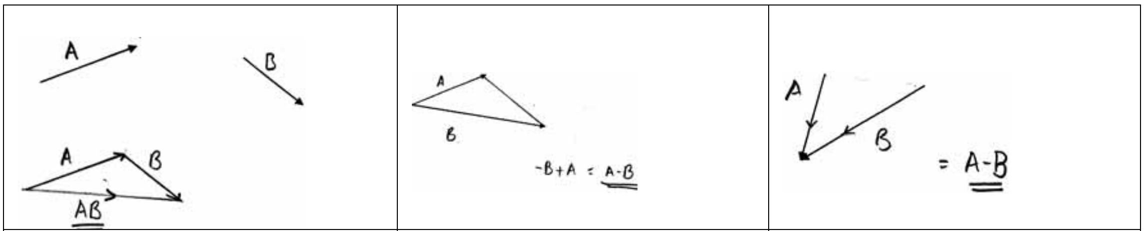
Comment

The student's answer raises an interesting point of what is considered to be mathematical response. The student said "Drawing the picture doesn't really answer the question." The implication is that graphical responses are not valued in Mathematics despite that the text book from the previous year included that stage of development in teaching vectors. The student did not seem to be aware that the numbers which she used did not have any significant value. In fact, as an answer, they were meaningless in both a mathematical and a physical sense. However the student was aware of the graphical responses she could give and gave a vague answer to the generic type of question (figure 9.5) and a much more precise response to the singular question (figure 9.6), as if the singular question demanded more thought from her.

It could be suggested from all of the student's responses that she understands vectors as a mathematical concept that can be used in the same way in any physical context. The student also does not mention the parallelogram law of addition and the only rule she mentions is 'nose to tail' movement, which is the necessary part of the triangle law of addition although this was never explicitly mentioned. She also does not use the commutative law of addition in graphical or symbolic mode.

9.2.3 Student S3

The third student (S3) answered most questions graphically. His pre-test responses to three of the questions are shown in figure 9.8.



(a) (b) (c)

Fig. 9.8 Student S3: examples of responses to the pre-test

The response in 9.8 (c) is equivalent to the graphical response as student indicates that to add vectors as journeys the direction of vector B would have to be changed and indicates it with '-B'. However the student actually completes the addition only in the question shown in figure 9.8 part (a). The relevant parts of the interview are shown below.

I: could you look at your answers and give me some idea of your thinking at that time?

S3: I thought it was asking me to put them into a triangle and then join up (fig. 9.8a).

I: What about the other two? (fig. 9.8 b and c)

S3: I did not know how to do it.

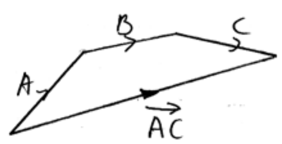
I: Was it anything you did in the past which made you answer in this way?

S3: I didn't know how to do it so I just guessed.

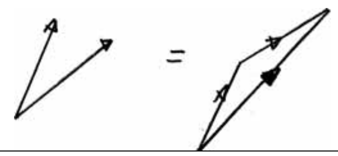
I: Is there any other way you could have answered any of these questions?

S3: I cannot think of any other way.

I: In question 5 you were asked to draw a representation of three forces and add them together and in question 6 you are asked to draw a representation of two displacements and add them together. Can you explain why you answered them in this particular way? (The student's answers are shown in figure 9.9.)



(a) response to Q5



(b) response to Q6

Fig. 9.9 Student S3: responses to different contexts questions in the pre-test

S3: I though that if they are displacements they have to come out from the same point (figure 9.8 (b)). [...] I thought that a displacement has to always start at the origin, hasn't it?

I: And what about forces?

S3: I just drew any three vectors?

I: No specific reason?

S3: No

I:: Why did you write $\overrightarrow{AC}$ here (the interviewer points to the resultant in figure 9.9 part (a)).

S3: I cannot remember. I think that's because it starts at A and end with the point on C. Should it be ABC?

Summary

The language student uses lacks clarity. For example when adding two vectors students says: "I thought it was asking me to put them into a triangle and then join up". What he probably means was to join the vectors and close the triangle. His explanation seems to indicate some learnt procedure and not the answer he thought out. He also mixes the names of vectors: displacement with position vector. Also the student's language of notation lacks precision. He does not realise that in symbol $\overrightarrow{AC}$ the A and C initially refer to specific points in space.

The student seem to concentrate on the procedure :" I thought it was asking me to put them into a triangle and then join up", which he thought is expected of him. It is not clear from the student responses if he focuses also on the effect of his actions. He was able to answer his own addition of two vectors (figure 9.9 part (b)) while he could not answer the same question set for him in a general context, as shown in figure 9.8 part (b). This could indicate that the context affects this student's thinking.

The student S3 does not show the flexibility in dealing with different modes. At no stage did he indicate that he could answer the questions symbolically/numerically, even when prompted. However this could just indicate that the student realised that the graphical mode was the most efficient way of answering the questions.

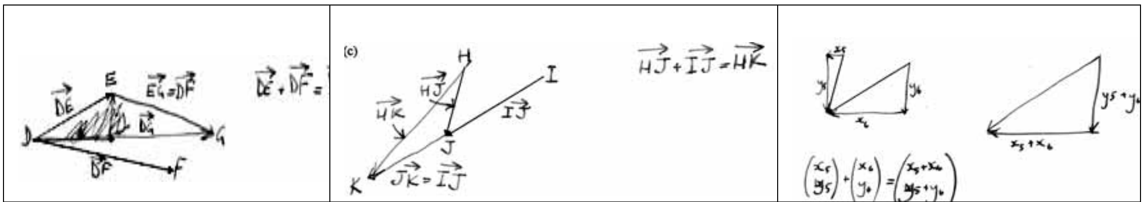
The student used a ruler to show all his responses, but at no time he used commutative law of addition or the parallelogram law of addition and only mentioned, in a vague way, the triangle as a way of adding. The student S3 seems therefore to be *high uni-modal focused* on the basis of some responses to the test.

Comment

One of the interviewed teachers suggested during the interview that students might feel that vectors drawn in this way are "fixed in space" and just "connected in the wrong way and simply join them up with a third vector." This might be a case with this student when the questions are set in the purely mathematical context. The student therefore lacks flexibility and versatility of using his knowledge. It has been not embedded properly nor turned into a cognitive unit. The student also mixes the notation of the free vectors, for example **u** and **v**, which can be written as a sum **u + v** with the notation of the displacement vector from A to B written as $\overrightarrow{AB}$ and from B to C written as $\overrightarrow{BC}$, which added together would give $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

9.2.4 Student S4

The next student (S4) has been categorised as ***fully integrated***. He could vary his answers from the graphical mode to symbolic and did not have any problems with different contexts or with the singular questions. Below are the examples of his test responses (figure 9.10) to the test.



(a) (b) (c)

Fig. 9.10 Student S4: examples of responses to the pre-test

I: What do you think the question is asking you to do? (refers to question answered in figure 9.10 part (a)).

S4: Find the resultant vector when there are two given together so if you put the end to end it will be an overall translation.

I: So what about the next one? (refers to question answered in figure 9.10 part (b)).

S4: The same. You add them together.

I: What do you think question 3 is asking you to do?

[Pause]

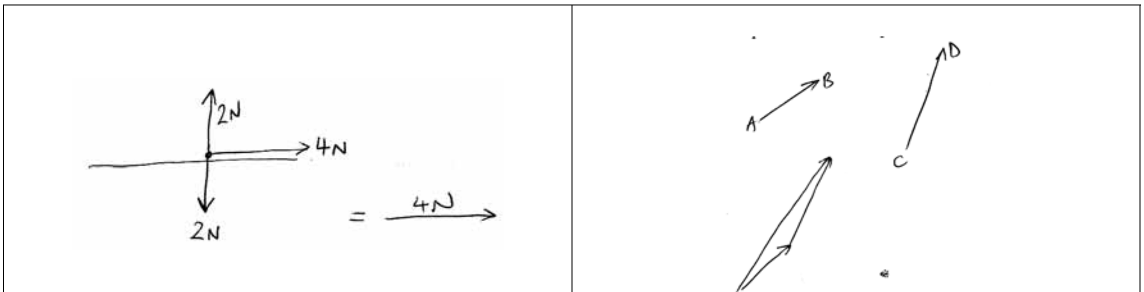
S4: If you display each vector into two perpendicular directions, and then add the two horizontal and the two vertical.

I: Did your previous teacher teach you this method?

S4: I don't think so, It makes sense, I must have got it from somewhere.

I: Thank you very much. Which method seems easier to you, the first one or the second one?

S4: If I was given values for the vectors and if they were given on the graph or squared paper I would find this one easier. [He points to figure 9.10 part (c)).



(a) (b)

Fig. 9.11 Student S4: responses to different contexts questions in the pre-test

I: Did they show you anything like this in Physics? (the interviewer points to the students' answers in figure 9.11 (a)).

S4: No I do not think [...] Yes we've done vectors as forces, so we would have used Newton's as the vectors. Stuff like … [He draws the example of three forces acting from one point with the values in Newton's next to them and angles in between, figure 9.12]

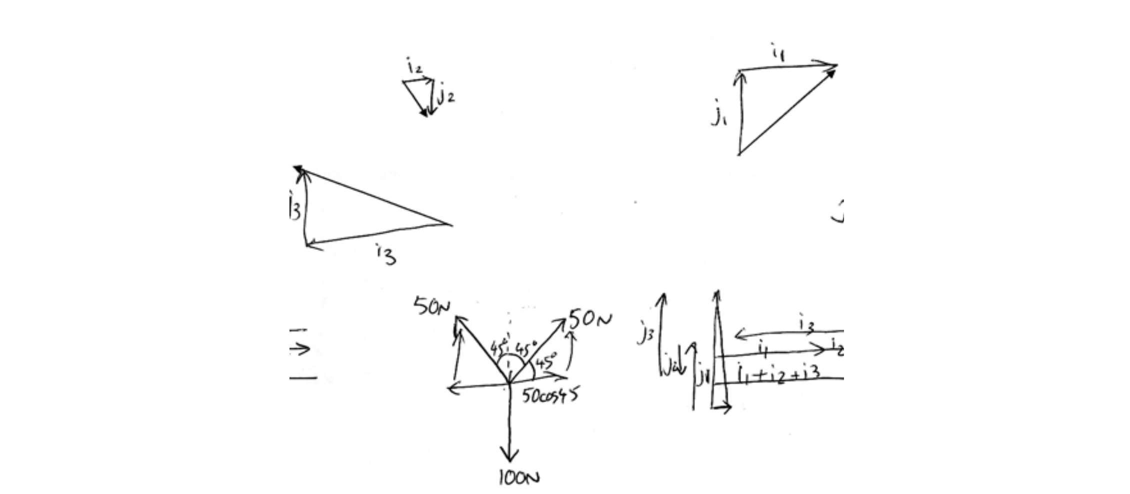


Fig. 9.12 Student S4: response to the interview question

I: How would you find the resultant?

S4: I would split them, yes, I would split into horizontal and vertical forces [draws horizontal and vertical component on each force] and add them.

I: Would you use the angles?

S4: Yes, this would be 45, [points] and this would be a hypotenuse [points to the force] then this would be cos 45° and so on.

I: What about the next one? [referring to the question shown in part (b) of figure 9.11].

S4: Oh, this is displacement.

I: Is that natural for you to draw the displacements separately?

S4: I just drew them like this because they are drawn separately in question 2.

I: But Question 2 says vectors not displacements?

S4: Well a vector looks more like a displacement. Displacements are obviously vectors.

I: So are forces vectors?

S4: Well they are. It is an arrow there (points to his answer with forces, to one of the vertical forces) but nothing actually moves.

Summary

The student S4 uses the language of vectors more flexibly than the previous 3 students. He uses phrases like: "if you put the end to end it will be an overall translation"; and "you display each vector into two perpendicular directions, and then add the two horizontal and the two vertical" (the word 'components' is missing from the last sentence). He also said: "I would split into horizontal and vertical forces" and does not maybe realise that they are only components of the forces and not two different forces.

From his statement about the "overall translation" we can also assume that he thinks about the effect of actions. He also considers forces acting on an object, without object actually moving although this is not quite correct according to his drawing in figure 9.12 as there will be a resultant force which would cause the object to accelerate.

The student S4 seems to think of a vector in the same way whatever the context. This could be implied from his verbal responses: "I just drew them like this because they are drawn separately in question 2"; "Displacements are obviously vectors; and in reply to a question "So are forces vectors?" he responded "Well they are. It is an arrow there". The student seem to understand (judging from his responses) the concept of vector as something representing a quantity which has a magnitude and a direction.

The student S4 seem to be able to operate in both modes (graphical or symbolic), in both generic and singular type of questions (figure 9.10 parts (b) and (c)). The student did not use the ruler to draw or measure but his drawings are fairly precise approximations and it is clear that he understands the idea of equivalent and free vectors. He does not use the commutative law of addition or the parallelogram law anywhere in his responses. The student was placed in the *fully integrated* category on the basis of the test and there is no evidence to change this categorisation.

Comment

The student seems to operate with ease on vectors using two different modes (graphical and symbolic) of operations, and in different contexts. He is aware of the same ***effect*** in a mathematical sense and does not try to use the vocabulary of a specific physical context when dealing with general situation: "Find the resultant vector when there are two given together so if you put them end to end it will be an overall translation." This student can deal with the singular cases (figure 9.10 (b) and (c)) and seems to adapt the mode of answering according to whichever is more suitable. It could be also concluded from some of the responses that student gained his knowledge and understanding of vectors on basis of one context: "Well a vector looks more like a displacement," and built this into a cognitive unit which he uses in other contextual situations. When asked if forces are vectors, he answers: "Well they are, it is an arrow there but nothing actually moves." The student is aware of the triangular law of addition but does not mention the parallelogram law.

9.3 The interviews following the post-test

The interviews were conducted with the student coded as:

- S5: from group B and classified as belonging to the *uni-modal* category;
- S6: from group B and classified as belonging to the *higher uni-modal* category;
- S7: from group B and classified as belonging to the *versatile* category.
- S8: from group A and classified as belonging to the *higher uni-modal* category;
- S9: from group A and classified as belonging to the *versatile* category;

There were no students in group A, at that stage, left in the *uni-modal* category.

9.3.1 Student S5

The examples of student's S5 responses are presented in figure 9.13. Part (i) shows two responses, the top one to question 2 (a) and the bottom one to question 3 (a). Similarly part (ii) shows responses to questions 2 (b) and 3 (b) and part (iii) shows responses to questions 2 (c) and 3 (c). The relevant parts of the interview with the student S6 are shown below the figure 9.13.

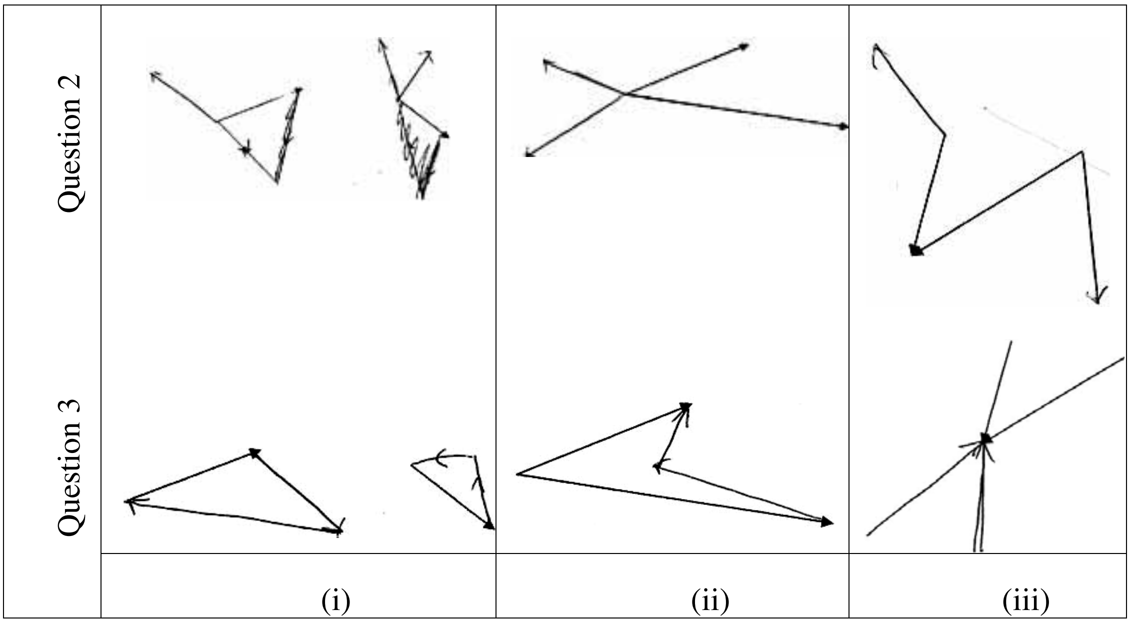
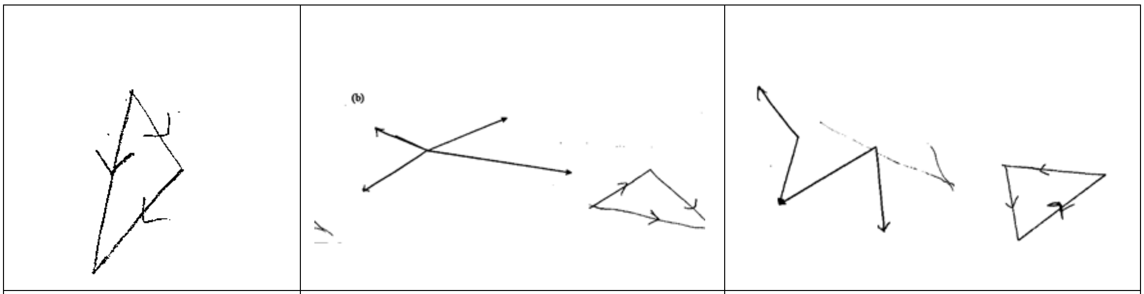


Fig. 9.13 Student S5: responses to questions 2 & 3 in the post-test

I: How did you answer question 2?

S5: I misunderstood the questions and I was adding another two vectors from the end of the one already there.

I: Can you answer the question as you understand it now?



(a) (b) (c)

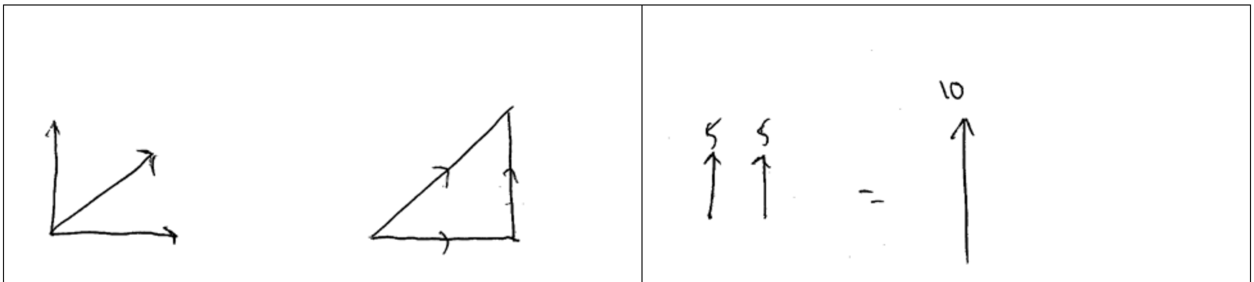
Fig. 9.14 Student S5: corrected responses to question 2 during interview

S5: [The student draws his answers as shown in figure 9.14.]

I: So what is the rule for adding vectors?

S5: When one ends, starts the other one, only I could not do it here [points to his answer in figure 9.14 (c)] because they meet in one point, but here I could because they start at one point [points to his answer in figure 9.14 (b)].

The student's S5 responses to the two questions, set in two different contexts, are shown in figure 9.15. Part (a) shows student's response when being asked to draw three different forces and add them together and part (b) when being asked to draw two displacements and add them together.



(a) - forces (b) - displacements

Fig. 9.15 Student S5: responses to different contexts questions in the post-test

I: How did you answer questions 5 and 6?

S5: I have seen questions like this before. [He points to his answer in figure 9.15a.]

I: Why did you draw answer in Q6 in this specific way? (referring to figure 9.15 (b))

S5: Because then they go in the same direction and I don't have to use sine or cosine, just add the two forces.

Summary

The student seems to say that he misunderstood the instruction "add the two vectors".

He also referred in both questions set in two different contexts to vectors as forces. When describing the rule for vector addition he said: "When one ends starts the other one" and never explicitly mentioned the 'nose to tail' rule.

He mentions two procedures he knows for the addition of vectors: one of them—in the graphical mode—seems to relate to him to vectors set in a general context, which he also tried to use in the case of adding forces (figure 9.15 (a)); however in case of adding two displacements, he mentioned a different procedure, "they go in the same direction and I don't have to use sine or cosine."

The student's interview answers, shown in figure 9.14 parts (a) and (b), lack the precision but show the correct concept of the vector addition, however in part (c) the resultant has the wrong direction. He is aware of the procedure of putting vectors 'nose to tail' when adding them graphically: "When one ends start the other one." However he could not add vectors in the singular case even after realising that adding vectors meant finding the resultant, but did not have the same problem with the question is generic for forces: "I could not do it because they meet in one point, but here I could because they start at one point."

He added forces (figure 9.15 (a)) into a closed triangle, without the resultant, even although his proper magnitudes did not agree with his assumption and therefore he does not seem to understand the equivalence of vectors. In the case of displacements he used a 1-dimensional situation for simplicity as he did not want to use "sine or cosine". He never used or mentioned use of the column vectors or other symbolic methods although by mentioning the trigonometrical ratios he is obviously aware of other methods of operation on vectors. He does not seem to be flexible in dealing with different modes of operation.

Comment

The responses of student S5, written as well as verbal, show his awareness of the idea of the '*same effect*' only at a very basic level. He did not use the flexibility of the language when discussing problems of addition of vectors and his knowledge is limited to the generic cases, which means that he has a limited procedural view. He lacked precision in his drawings, and therefore had no awareness of operating on equivalent or free vectors, which might have influenced his development of conceptual ideas. He did not have symbolic knowledge to fall back on either. He seem to be a good example of what the lack of teaching proper techniques of drawing equivalent vectors can cause and how it can prevent building the concept of vector into a cognitive unit. This student does not seem to be aware of the parallelogram rule of addition and is only aware of the rule of joining the beginning of one with the end of the other and thinks that 3 forces should make a triangle so he does not understand the idea of the resultant force. He also does not give any indication of awareness of the commutative law of addition.

After the interview the student was still classified in the ***uni-modal*** category. The student answers only graphically but at a lower stage of the cognitive development.

9.3.2 Student S6

The responses of student S6 to questions 2 and 3 are presented in figure 9.16. Part (i) shows two responses, the top one to question 2 (a) and the bottom one to question 3 (a). Similarly part (ii) shows responses to questions 2 (b) and 3 (b) and part (iii) shows responses to questions 2 (c) and 3 (c). The relevant parts of the interview with the student S6 are shown below the figure 9.16. In each case he draws the resultant as a line of dashes.

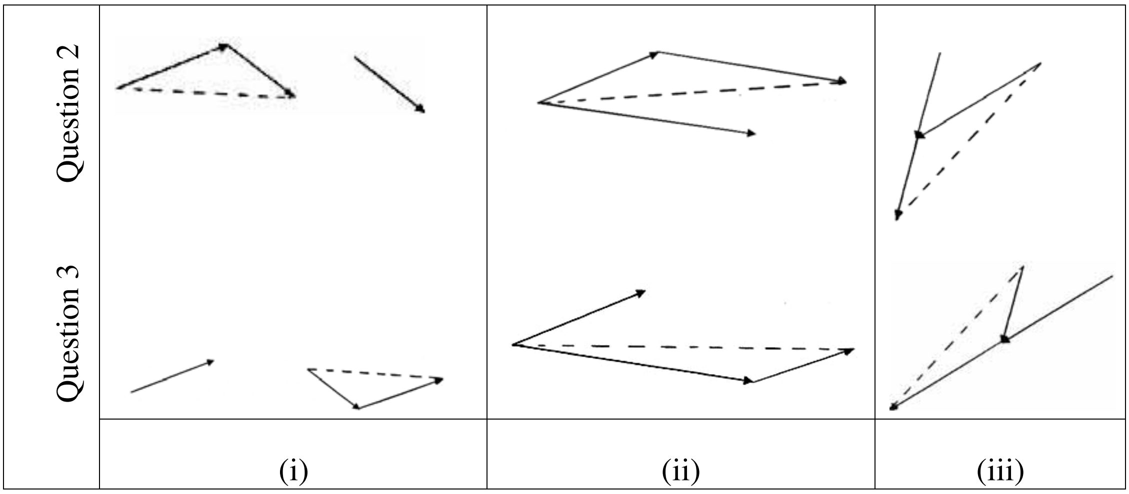


Fig. 9.16 Student S6: post-test responses to questions 2 & 3

I: Can you talk me through your answers to question 2?

S6: I can do it in two ways [indicates with the pencil translating vectors to the end of one another in two ways].

I: what about b and c?

S6: The same [indicates with the pencil translating vectors to the end of one another in two ways].

I: What about question 3?

S6: Simply moved vectors in a different order.

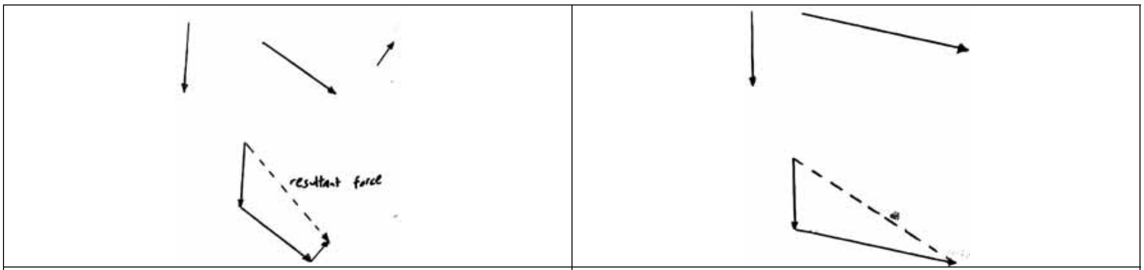
I: Did you think at all about answering in a different way.

S6: I could draw horizontal and vertical but it would difficult in this case.

I: If you go back to the question 2. How did you choose the direction of the arrow for the resultant vector?

S6: It is simply the direction of the two arrows together.

The next figure (9.17) shows the responses from student S6 to the two questions set in two different contexts.



(a) (b)

Fig. 9.17 Student S6: post-test responses to the different contexts questions

I: In this question 7 (figure 9.17 part a) you have to represent 3 forces and add the together, how did you do it?

S6: It is the same as adding three vectors together.

I: Why did you specifically draw vectors like this? Is this all right for forces?

S6: Well, these are just representations of forces.

I: What about question 8?

S6: First you move in this direction [points to one of the separate vectors] then in this direction [points to the other separate vector] then you add them together.

I: It does not worry you that they are not connected?

S6: We can transfer vectors anywhere.

Summary

The student S6 indicates, in the language he is using, that he treats vectors in the same way whatever the context. However looking at his graphical responses he does not put arrows on the resultant vector, even when prompted, anywhere apart from the question set in the context of forces (figure 9.17 (a)). He implies the commutative law of vector addition when he says: “Simply moved vectors in a different order”. He also implies that he realises some idea about the same *effect* by saying “It is simply the direction of the two arrows together”. However, he only mentions the direction and not the magnitude. He realises that vectors are only the “representations of forces” and displacements and indicates that he has a concept of equivalent and free vectors by saying: “We can transfer vectors anywhere”.

The student S6 seems to be focusing on the effect of the procedures and treats vectors in the same way whatever the context. He also implied (“I could draw horizontal and vertical, but it would difficult in this case") that he is aware of the symbolic mode of operation and did not use it as, according to him, it is not proper in this case. This could mean that he has flexibility in dealing with different modes of operations and uses the most appropriate one for the question.

Comment

The student S6 seem to have very good grasp of a vector as a mathematical tool for solving problems in mathematical and physical problems. He is very competent in his use of the equivalent vectors and free vectors and realises that the commutative law of addition applies to addition of vectors. What is not clear is how he understands the idea of the same *effect*. He seems to understand it in the context of forces. His resultant force has a direction but the rest of the questions seem to lack that part of the answer. It might be possible that his understanding of the addition is limited to following the arrows from the beginning to the end, after they are placed 'nose to tail'. Further questioning was not possible due to the lack of time.

The student S6 maybe could be reclassified as versatile, but the classification is unclear as he never explains why he does not place the arrows on his resultant vectors in response to the addition.

9.3.3 Student S7

The next interview was conducted with another student from group B who was coded as S7. This student was also classified to belong to the ***multi-skilled*** category according to his responses to the post-test and observation of how he was attempting the test questions (he was measuring horizontal components and angles of the vectors, adding them together and then drawing the resultant vector, which is indicated in oblongs on his responses). Some of his answers to the post-test are shown in figure 9.18. Part (i) shows two responses, the top one to question 2 (a) and the bottom one to question 3 (a). Similarly part (ii) shows responses to questions 2 (b) and 3 (b) and part

(iii) shows responses to questions 2 (c) and 3 (c). The relevant parts of the interview with the student S6 are shown below the figure 9.18.

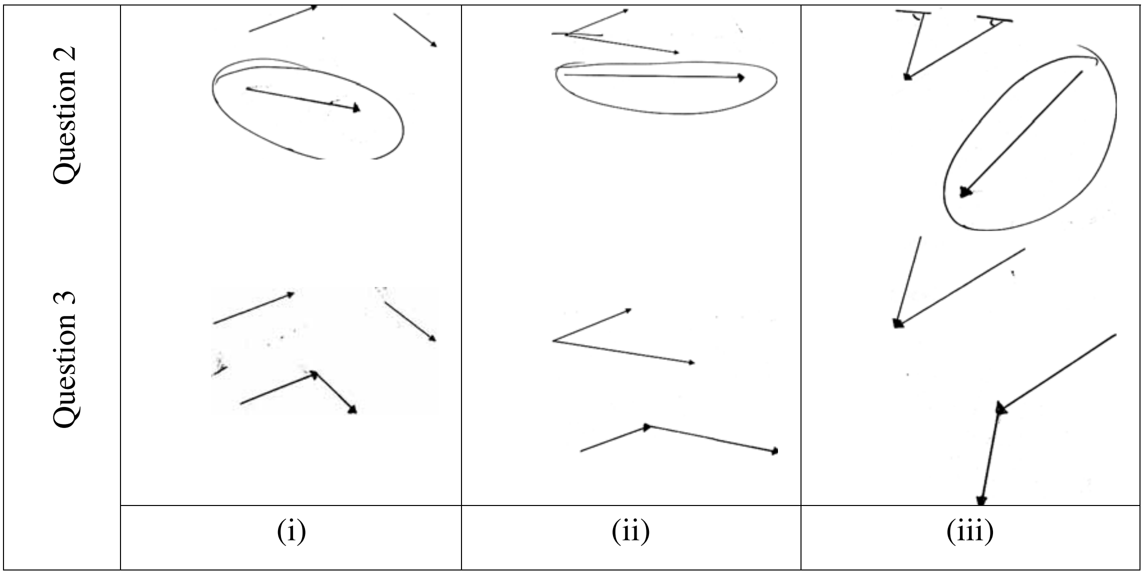


Fig. 9.18 Student S7: responses to the pre-test questions 2 & 3

I: How did you answer question 2?

S7: I used a ruler and compasses [he means protractor] to measure these vectors and then drew them here [points to the answers in oblongs].

I: Can you show me what you did?

S7: I very roughly took an angle [uses protractor to measure the angles from the horizontal direction] so it is twenty and then the other one here, which is about 40 and drew them together and measured the answer...and....[shows the resultant vector].

I: What about question 3?

S7: The problem with this question ... I did not know what you meant by "add the two vectors" so I assumed it was put them together as arrows.

I: What do you understand by the addition?

S7: Join them by end to tail and draw the arrow between them.

I: When we look at your answers to the next question you actually put them as we say 'nose to tail' but you did not draw an arrow?

S7: Well I did not quite know what add means so I just join them together so it shows the direction.

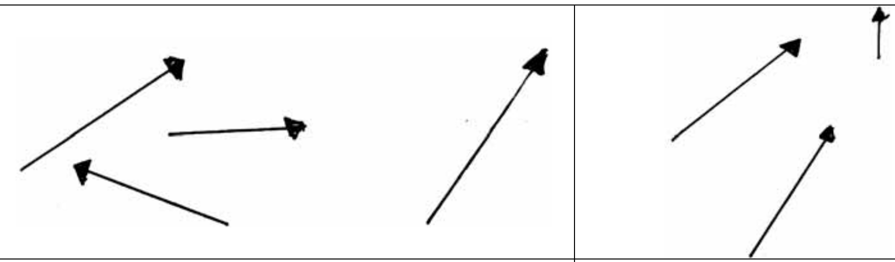
I: So do you understand addition of vectors as putting them together?

S7: Yes I understand the addition as showing the total movement.

I: So did you show the total movement in the previous question?

S7: Yes, and here I am showing the total movement but in two separate parts. If I was told to put vectors together I would draw the resultant force or whatever movement it was and the other way I just thought I would show it the other way.

Figure 9.19 shows the student's S7 responses to two different contexts questions: part (a)- displacement; part (b)-forces.



(a) (b)

Fig. 9.19 Student S7: responses to the questions set in different contexts

I: What did you do in the question 7 and 8?

S7: Yes, the same thing.

I: Can you explain, why did you answer both questions in the same way?

S7: Apart from that there is an extra force in 7, they are exactly the same.

I: Why do you think they are the same and yet there is a different physical situation?

S7: How do you mean a different physical situation?

I: The first refers to three forces and the second to two displacement

S7: I honestly did not read it like this. Ah,…… The forces ……..are not necessarily vectors I don't think, they are movements. Whereas displacements are distance from a point, displacements and vectors are different things, the displacement is a distance from a point, which will be still the same. Sorry I will start again. These are vectors (points to the question on forces) and have magnitude and direction, whereas these are just movements from a certain point, it is just a difference, a movement, which is this one.

Summary

The student's S7 use of language when discussing problems connected with vector addition is confused, especially when he talks about addition of vectors representing the physical quantities (forces and displacements). His last response indicates it very clearly. He answered all his questions not thinking about the contexts they were set in but when this was brought to his attention he was confused.

His focuses on vectors as an action of movement ("If I was told to put vectors together I would draw the resultant force or whatever movement it was"), even when he has to deal with forces he think of movement ("The forces ……..are not necessarily vectors I don't think, they are movements").

He implies that the addition can be shown in two different ways: one when he shows the resultant "I understand the addition as showing the total movement"; and another as a journey following the route, without showing the resultant "I am showing the total movement but in two separate parts".

The context he is working in makes a difference to the way he thinks: "These are vectors and have magnitude and direction, whereas these are just movements from a certain point, it is just a difference, a movement.

He can use two modes of operation, however only at a lower stage of the cognitive development (journey) in the graphical mode.

The student was classified as *multi-skilled*; maybe he could be classified as *higher uni-modal* in the symbolic way, but this is also arguable.

Comment

The student S7 displaced great confusion in his understanding of vectors and vector addition. He has answered some further questions in a way which could indicate that he could be classified as *higher uni-modal* on the basis of all of his test responses but not on the responses shown above. His language lacks flexibility. He did not develop a concept of vector as a cognitive unit.

9.3.4 Student S8

Some students in group A were inconsistent in their graphical solutions and one of them, S8, was interviewed to find out how serious the problem was. The student was classified as ***multi-skilled***. For example, as shown in figure 9.20, the student did not draw the resultant in some general addition questions. As previously, some of his answers to the post-test are shown in figure 9.20. Part (i) shows two responses, the top one to question 2 (a) and the bottom one to question 3 (a). Similarly part (ii) shows responses to questions 2 (b) and 3 (b) and part (iii) shows responses to questions 2 (c) and 3 (c). The relevant parts of the interview with the student S6 are shown below the figure 9.20.

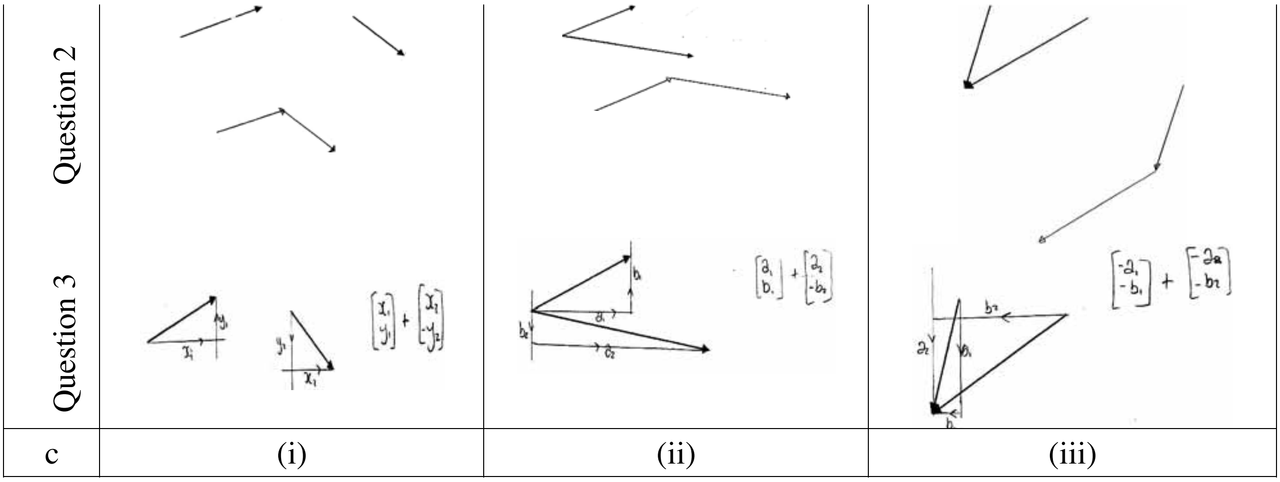


Fig. 9.20 Student S8: post-test responses to questions 2 & 3

I: When you were answering question 2 did you give me the full answer?

S8: As I was meant to. I could do it differently now. I could draw an arrow from here to here [showed the correct resultants with his hand].

I: So that would be your alternative?

S8: Yes.

I: Do you remember why you did not draw the arrows?

S8: I must have read the questions the wrong way?

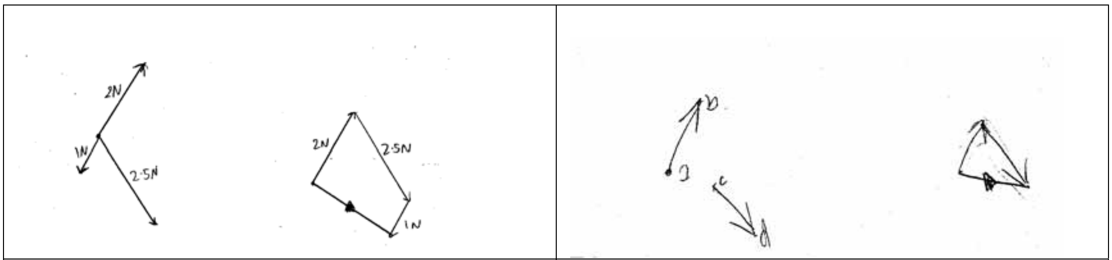
I: Can you explain a bit more?

S8: If it said show the resultant I would have drawn an arrow going from that point to that point [shows correctly with the pen]. When you added vectors you do it in a different way.

I: In the next question you showed two column vectors but you did not give the final answer, why?

S8: It is the same, because it shows the direction.

In the next part of the interview the student was asked to complete the questions on vectors in different context that he missed out in the test, therefore figure 9.21 (a) shows the questions completed during the interview. The next question (figure 9.21b) was done from the beginning during the interview.



(a)	(b)

Fig. 9.21 Student S8: responses to the post-test questions set in different contexts

I: In question 7, if I had asked for the resultant, how would you draw it? (Fig. 9.21a).

S8: Should I draw it?

I: Yes please.

S8: [Student drew the correct resultant.]

I: When I asked the next question you did not answer, was there any reason for it?

S8: I don't know.

I: If I asked you to do it now, how can you do it?

S8: [student's work in Fig. 9.21b].

I: When you drew the forces you drew them from one point, the displacements you drew separately, can you explain why?

S8: I kind of relate forces acting on a point, whilst a displacement I don't know, I would see it as some type of instrument.

I: How do you see displacement?

S8: Moving an object…..across a distance.

I: If you would have to make a story what story could you make?

S8: If someone would walk from *A* to *B* and then from *C* to *D*, how far would they walk, I don't know [points to his triangle] I don't know. If they were together [draws them following each other] it would be easier to explain. I don't know why I drew them separately.

Summary

This student in the same was as some of the previous students connects 'adding vectors' with placing them 'nose to tail' and nothing else, but when asked to give the resultant seems not to have a problem. He also mentions that adding vectors is showing a direction, which could imply the total effect of the addition.

The vectors set in a context of displacement, seem to make a difference to the way he was thinking, but only when he actually is prompted to think of that context in a precise way. His first impulse was to just treat displacements as any vectors. However when dealing with the context of forces he was he was very precise of the way he was thinking: "I kind of relate forces acting on a point".

He indicated that, as long as the question asks for the resultant and not addition, he can manipulate vectors in both modes of operation, even in case of the singular questions. He therefore remains in the *multi-skilled* category.

Comment

The student, coded S8, seems to be confused in the language of vector addition. If the vectors are journeys in his mind then he might be thinking that the addition means showing whole 'journey' from the start to the end (figure 9.20). However this is not consistent with his other responses (figure 9.21). This student is from the experimental group and drew the displacements separately in the test (figure 9.21b) but could not remember why. So although he might have remembered from the experimental lessons that the displacement can be drawn separately he did not built a cognitive unit of a free vector from that work and therefore moved only partly towards understanding. Maybe he would have benefited from revisiting the idea a few times.

This can be triangulated with the teachers' comments. One of the teachers said that the adding vectors is putting them 'nose to tail' but because the answer is 'obvious' some of them omit showing it, which could have been one reason for omitting indicating the 'total effect'. However, in case of the student S8, the omission could be due to lack of understanding of the language rather that not realising the concept of the total effect. He seems to be connecting addition with the procedure of placing vectors 'nose to tail', and showing the resultant with the total effect of 'addition'.

9.3.5 Student S9

The student coded S9 was chosen as she was one out of six students in group A, who, in the post-test analysis, was classified into the *versatile* category. The interview meant to check whether the student is indeed flexible in her thinking and whether she just uses the procedures or whether she has a conceptual understanding of vector addition. Figure 9.22 shows the student's responses to question 2 & 3, which asked to add vectors in two different ways. Part (i) shows two responses, the top one to question 2 (a) and the bottom one to question 3 (a). Similarly part (ii) shows responses to questions 2 (b) and 3 (b) and part (iii) shows responses to questions 2 (c) and 3 (c). The relevant parts of the interview with the student S6 are shown below the figure 9.22.

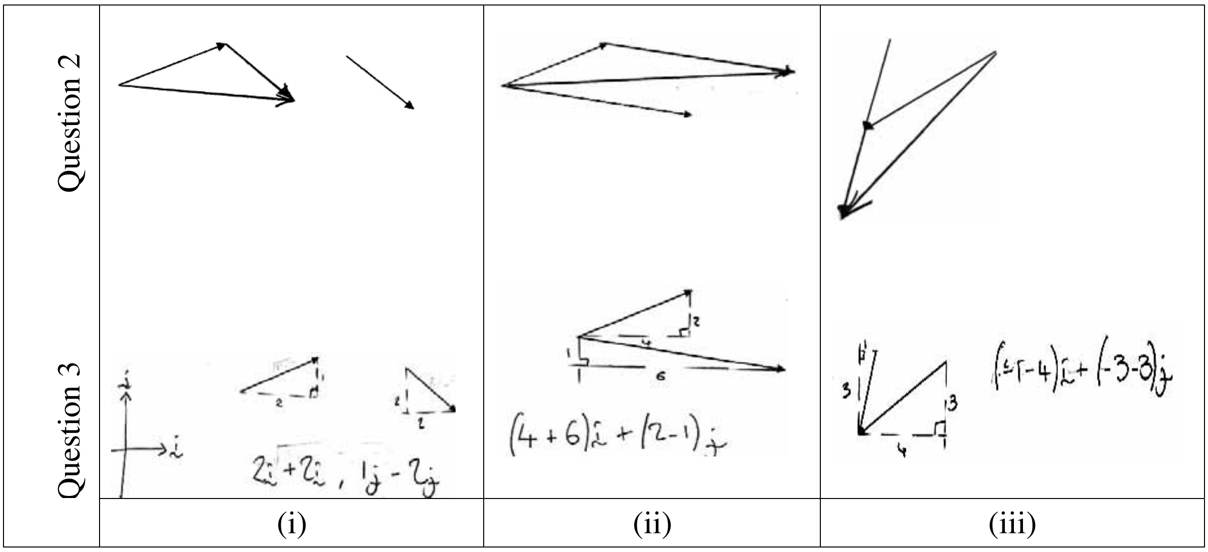


Fig. 9.22 Student S9: responses to the post-test questions 2 and 3

I: How did you answer question 2?

S9: I was sliding the vectors so one is at the end of the other one, so that they are nose to tail, and then drew a resultant from the beginning of one to the end of the second one (shows correctly for all three with her finger)

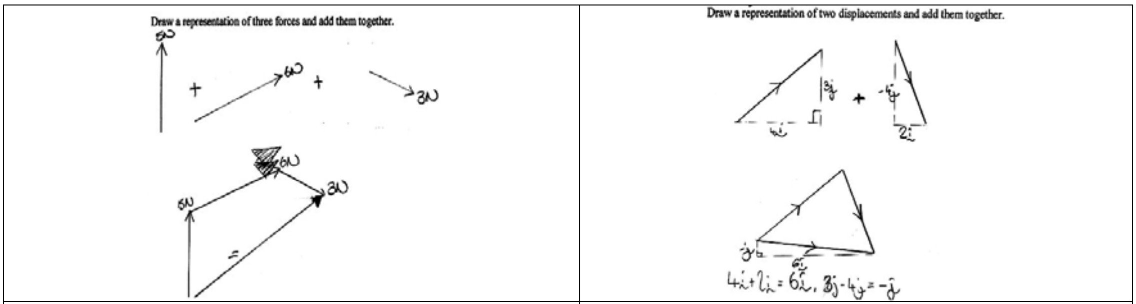
I: So what about the next question 3?

S9: I worked out the length and direction and put them together. I did them in i and j directions and added them together.

I: Did you do questions like 2 (c) before?

S9: I've never done questions like this before, so I was making my own way of doing it.

The responses from student S9 to the two questions set in two different contexts are shown in figure 9.19. Part (a) shows the student's response when being asked to draw three different forces and add them together and part (b) when being asked to draw two displacements and add them together.



(a) (b)

Fig. 9.19 Student S9: responses to the post-test questions set in different contexts

I: Could you look at questions 5 and 6? (figure 9.19 a and b)

S9: They are the same. You could do them in i and j directions and add them together or you could draw them so they are nose to tail and draw the resultant.

Summary:

The interview indicated that the student S9 reached the highest stage of cognitive development in both graphical and numerical mode. He gives enactive responses like: “slide the vector”, “put them together”, and “I drew them so the tail of one is at the beginning of that one” and after reflecting on his ‘actions’ could this through the next stage of the development The student is not confused about the language of addition: “I did them in i and j directions and added them together” or the total effect: “then I drew a resultant from the beginning of one to the end of the second one”. She was not confused by the two different contexts and said: “They are the same. You could do them in i and j directions and add them together, or you could draw them so they are nose to tail and draw the resultant.” By the same statement the student implies its own flexibility in using either modes of operation. It seems from this student’s responses that she can use the procedures very well and uses mathematical concept of vector in different contexts. In further questions in the test the student also used the commutative law of addition.

Comment:

The student S9 showed the ability to operate at the highest cognitive stage of development in both modes of operations. She showed flexibility in dealing with singular cases "I've never done questions like this before, so was making my own way of doing it." When asked to comment about two questions placed in different contexts his response was: "They are the same." which indicates that she used the mathematical concept of vector independent of the context. She used the triangle method of addition but did not mention or imply the parallelogram law. The student is still considered to be at least in the ***versatile*** category or maybe even *fully integrated*.

9.4 Summary from the interviews

By performing the triangulation between the responses from the teachers and the responses from the students we can see that, although the mathematics teachers anticipated students' perception of vector addition as thinking of compiling journeys, the students at higher levels of cognitive development did not have this problem. They developed a concept of vector as a cognitive unit which they could use flexibly.

The students working at the lower levels of the cognitive development had problems/misconceptions in their responses to both the generic and singular questions which were anticipated by the mathematics teachers. For example the student S1 answered the question in figure 9.1 (a) as if the vectors were "fixed in space." The teachers also anticipated that students might not show the resultant as "they might feel the addition means placing vectors one after another," and indeed student S1 (figure 9.2 (b)), student S7 (figure 9.18) and student S8 (figure 9.20) had that problem.

They also thought that students might have more problems with singular questions and we can see this happening in case of the student S3 who answered the simpler generic question (figure 9.8 (a)) but had a problem with more complicated generic question and the singular question (figure 9.8 (b) and (c)). The same situation occurred with student S5 (figure 9.13).

The Mathematics teachers were also saying that questions which might evoke physical implications may cause problems; this occurred with students at the lower stages of the conceptual ladder. This was especially clear in the questions set in two different contexts. Students S1, S3 before the course and S5 afterwards showed clearly that their thinking was changing dependent on the context and also with questions which might imply a physical context (two vector starting at one point, as in questions 2 (b) and 2 (c)).

On the other hand the problems anticipated by the Physics teacher that students will not be able to add vectors drawn separately occurred only in case of the student S1 and only in one question (figure 9.1 (a)). However her prediction that the students might not place arrows on the resultant vectors proved correct in many cases and one of them occurred with the student S6 (figure 9.16), who did not correct this omission even when prompted in the interview.

It seems from the interviews that the students could not communicate their problems as clearly as the teachers in anticipating the problems and therefore the interviews may not, in some cases, show the students' misconceptions or strengths clearly enough.

Generally all of the students interviewed concentrated on specific procedures, however the students working on the higher levels of the cognitive development generally made better connections between those procedures. Although different contexts affected most of their thinking, the higher level students seem to have been using a vector as a tool to solve problems of addition, showing the awareness of both aspects of a vector: magnitude and direction. The lower level students seemed to have concentrated only on one of those aspects (mainly direction) and ignored the other one (magnitude). This gives support to hypothesis 3 that the students operating at the higher levels conceive the concept of vector as a cognitive unit, as an entity in itself which can be used in different contexts.

In chapter 7, the question arose as to whether students were revealing their full ability in the separate graphic and numeric modes on the questionnaire. The students who answered questions mainly symbolically but at a higher level, as seen in the interview with the student S2, proved to be *multi-skilled* and capable of answering questions graphically at a higher level. However this does not necessarily follow with students who responded only graphically at a lower level. Students S1, S3 and S5 did not show this ability, even when prompted to respond symbolically.

There is an apparent difference in the language which the students operating at different levels of cognitive development use to describe their responses to the test questions. This difference occurred in the interviews following the pre-test as well as in the interviews following the post-test.

The lower level students use phrases like: "I connected all the vectors together so it will be easier to add them all together," (S1); I did not draw them to scale but to different scale just to give a general idea of how to add them," (S1); "you can go shorter distance from A to B," (S1); " I thought it was asking me to put them in a triangle and then join up," (S3); "I cannot think of any other way," (S3); "When one ends starts the other one," (S5); "Well, I did not quite know what add means so I just join them together so it shows the direction," (S7); "... displacements and vectors are different things...,"(S7).

The higher level students use phrases like: "I put them nose to tail," (S2); "... if you put the them end to end it will be an overall translation," (S4); "If you display each vector in two perpendicular directions, and then add two horizontal and the two vertical," (S4); "Displacements are obviously vectors," (S4); "simply moved vectors in a different order," (S6); "Well, they are just representations of forces," (S6); "I was sliding the vectors so one is at the end of the other one, so that they are nose to tail, and then drew a resultant from the beginning of one to the end of the second one," (S9); "They are the same. You could do them in **i** and **j** directions and add them together or you could draw them so they are nose to tail and draw the resultant," (S9).

From the responses we may conclude that the higher-level students were more likely to develop the concept of vector as a cognitive unit, while the lower-level students were not. This gives evidence to support hypothesis 3 from a qualitative viewpoint, giving fuller information to underline the quantitative support in chapter 7.

The interviews were consistent with the overall theoretical framework, revealing new detail. For example, when students connect vectors ‘nose to tail’, they do not use the idea of free vectors, only the procedures of joining different journeys together to show a total journey. They therefore do not add vectors in the mathematical sense. Another important aspect is that these students do not consider the parallelogram law when adding two vectors and that the triangle law has an overpowering importance with some of them to such an extent that they do not use equivalent vectors.

9.5 Overall triangulation between the interviews and the quantitative data

The interviews in this chapter revealed a consistency between the teachers’ views of the kind of difficulties that the students would have with the questions and the responses of the students. There is a clear difference in the views from physics and mathematics, where the first focuses on meaningful real-life examples which, nevertheless cause difficulties with the concept of free vector, and the latter focuses on the development of the concept within successive years of the syllabus.

The interviews in this chapter were consistent with the idea that the quantitative study satisfactorily represented the students’ performance, with the exception that some graphical questions may not show the full range of symbolic thinking that was available to the student as this was not directly required. This confirmed the decision to measure the higher stages attained by the students rather than taking a numerical average of student performance across the whole set of questions.

The language of the students at different levels revealed graphically that students who were succeeding at the higher levels regarded the notion of free vector as a coherent single concept that had meaning across different contexts whereas the students who were less successful tended to apply different procedures in different contexts. The fluent and flexible way in which the more successful students operated with the concept of free vector is consistent with hypothesis 3 and gives support the general theoretical framework. The research hypotheses formulated in chapter seven are supported by the evidence, both quantitative and qualitative.